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A shared vision of effective teaching and learning of K-8 Mathematics in the Jefferson County Public Schools

An effective and coherent mathematics program should be *guided* by a clear set of content standards, but it must be *grounded* in a clear and shared vision of teaching and learning – the two critical reciprocal actions that link teachers and students and largely determine educational impact. While curriculum, materials, professional development, assessment and cultivating broad programmatic support are all necessary components of the educational enterprise, they have little real impact unless they are effectively enacted in each and every classroom where learning is facilitated, supported and maximized.

Accordingly, to ground and guide the development and implementation of a highly effective school mathematics program for *all* students, we describe a research-based vision of teaching and learning with twelve interrelated characteristics of effective instruction in mathematics. It is hoped that this vision will define and inspire excellence in every Jefferson Country Public Schools classroom where mathematics is taught and learned.

Effective mathematics instruction is thoughtfully planned. An effective lesson provides multiple opportunities for student learning and must be carefully planned. The days of minimalist lesson plans like "examples 1 and 2 on page 154" or "lesson 6-4: vocabulary, discussion, practice, homework" do not adequately reflect the demands and expectations teachers face. Rather, prior to teaching a lesson, teachers should be empowered and expected to:

- have a clear understanding of the specific learning expectations for their students and how and where these expectations fit in to the larger instructional unit;
- select and try out the set of problems, tasks and/or activities that support the specific learning expectations;
- identify a set of key questions and considered the required explanations that support the problems, tasks and/or activities to be used;
- consider the likely errors that students are likely to make and misconceptions that students are likely to have, and prepare strategies that address these errors and misconceptions; and
- identify the means by which the degree of student learning will be determined.

The heart of effective mathematics instruction is an emphasis on problem solving, reasoning and sense-making. Nearly every survey of business and industry addresses the critical need for current and prospective workers to be able to reason, question and solve problems. Thus the focus on problem solving as the heart of mathematics and on inquiry as the heart of science are societal, as well as educational, imperatives. However, beyond just rhetoric, effective instruction must consistently include opportunities for students to formulate questions and problems, make hypotheses and conjectures, gather and analyze data, and draw and justify conclusions. This is why students in effective classrooms regularly encounter questions like "why?", "how do you know?", "can you explain that?"

Effective mathematics instruction balances and blends conceptual understanding and procedural skills. Real mathematical literacy is as much about understanding the concept of division, knowing when and why to divide, and being able to interpret the meaning of a remainder as it is about merely knowing how to use an algorithm to find a quotient. Too often, the focus of instruction is on the one right way to get a single right answer, at the expense of understanding why this is the appropriate mathematics, how it relates to other mathematics, and when such mathematics should be used. For this reason, effective instruction balances a focus on conceptual understanding (e.g., the meaning of area and perimeter and how they are related) with a focus on procedural skill (e.g., how to find the area and perimeter of plane figures).

Effective mathematics instruction relies on alternative approaches and multiple representations. At nearly any moment in nearly any class, we know that many students are not processing the content in the way the teacher is processing the content. For example the teacher may be visualizing "three-quarters" as three out of four slices of a small pizza, while one student "sees" three quarters or 75 cents, another student "sees" three red balloons out of a total of four, and still another student "sees" three-quarters of an inch on a ruler. Effective instruction recognizes that students conceptualize mathematical and scientific concepts in different, but often equally appropriate, ways. Effective instruction incorporates deliberate attention to such multiple representations, including concrete materials, and to alternative approaches to accommodate the diverse of learning styles within every class.

Effective mathematics instruction uses contexts and connections to engage students and increase the relevance of what is being learned. Teachers have a choice. They can rely on abstractions and rules that are rarely connected to realistic situations or common contexts and ask students the equivalent of finding F when S = 81 in the function $F = 4 (S – 65) + 10$. Or teachers can take these abstractions and embed them in realistic contexts and problem situations that bring the mathematics and science to life. In this example, telling students that the speeding fine in a particular state is "$4 for every mile per hour over the 65 mph speed limit plus a $10 handling fee for the Police Department" and asking first for the fine when a driver is going 81 mph and then determining a driver's speed if they received a fine of $102. Then consider using a graphing calculator or computer software to represent this function in a table and a graph as well as symbolically, showing where and how the "point" (81, 74) exists within each representation.

Effective mathematics instruction provides frequent opportunities for students to communicate their reasoning and engage in productive discourse. The active, engaged, thinking classroom is a classroom of questions and answers, of inquiries and explanations, of conjectures and justifications, and of written and oral discourse. We know that writing helps to clarify our thinking and that teaching another strengthens our own learning. That is why effective classrooms are often vibrant environments of student communication in the form of explanations, dialogues, arguments and presentations.

Effective mathematics instruction incorporates on-going cumulative review. Almost no one masters something new after one or two lessons and one or two homework assignments. That is why one of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson. Teachers do this as part of a daily warm-up or as "bell-work" that focuses on recent instruction or as a daily "mini-quiz" containing

4 to 6 problems that keep skills sharp, review vocabulary and reinforce conceptual understanding.

Effective mathematics instruction employs technology to enhance learning. Calculators, computers and scientific instruments are increasingly important tools for supporting learning and making instruction more relevant. Graphing calculators that link symbolic, tabular and graphical representations of functions help students develop critical understandings of algebra. Geometry software and scientific simulation software enable students to create mathematical and scientific environments and analyze the impact of changes in selected conditions. Electronic blackboards significantly enhance the impact of such software. But it is not the mere use of technology that enhances learning, any more than it is the use of manipulative materials that "teach". Rather, it is the appropriate, planned and deliberate use of technology to support the development of mathematical understanding that impacts learning.

Effective mathematics instruction maximizes time on task. Videos of classes are striking in their variation in the number of minutes of engaged time on task, that is, the use of time for activities that engage learners and support learning. Some classes begin even before the bell rings with warm-up work posted or distributed at the door, use efficient and established routines to go over homework, transition smoothly from one segment of the lesson to another, keep the focus on student work and student thinking for extended periods of time, are rarely interrupted, do not confuse class work with homework, and still allocate time to building positive relationships with students. In this way, a 45 minute class can easily provide more than 40 minutes of engaged time on task. Other classes, when observed or captured on videotape reveal as much as 20 or more minutes frittered away with organizational matters, frequent interruptions, poor transitions and off-task chatter.

Effective mathematics instruction uses multiple forms of assessment and uses the results of this assessment to adjust instruction. When our focus shifts from what was taught to what was learned, the focus must also shift to assessing what has been learned. While tests and quizzes will continue to be important components of assessment, it is how the results of these quizzes and tests are used to assess the impact of teaching, plan reteaching, prepare individual instruction and design additional diagnosis that translates assessment into better teaching and learning. In addition, effective teachers use observations, class work, projects, and similar vehicles to monitor the quality of learning. Finally, the results of a carefully aligned system of unit tests and end of grade and end of course assessments are regularly analyzed to make curricular and instructional modifications.

Effective mathematics instruction integrates the characteristics of this vision to ensure student mastery of grade-level standards. The often elusive goal of assisting all student to achieve mastery requires a coherent and supportive program that aligns a vision, a set of standards, instructional materials, assessments, collegial sharing and professional development. Moving from mastery by some to truly ensuring mastery by all requires shifts in mind-sets to align with this vision, shifts in curriculum expectations, shifts in instructional practices, and shifts in allocations of resources. It requires a deep commitment to quality and a non-negotiable belief that all students can learn mathematics.

Effective teachers of mathematics reflect on their teaching, individually and collaboratively, and make revisions to enhance student learning. Finally, effective teachers replay their instruction, reflect on what appeared to work and what was more problematic, and examine student responses and work as part of an ongoing cycle of plan – teach – reflect – refine and plan all over again. Moreover, effective teachers work collaboratively with colleagues on issues of the mathematics embedded in the instructional tasks that are used, the pedagogical features of the instruction we conduct, and the student learning evidenced by analysis of student work.

On the one hand, we know with certainly that the elements of this vision do not get implemented by exhortation. We know that people will not do what they cannot envision and cannot do what they do not understand. And we know that the lack of a clear and shared vision commonly results in incoherent, often conflicting, policies and a widely shared perception that "this too will pass."

On the other hand, we also know that once a broadly shared understanding and acceptance of the elements of this vision of effective teaching and learning are in place, school districts, schools, teachers and parents then have a common platform upon which to organize, structure and improve a high quality mathematics program.