**SPEAKER:** Dr. Wu is a professor of mathematics at the University of California, Berkeley, where he taught from 1965 to 2009. He was one of the writers of the common core standards, and is currently serving on the TIMS 2011 science and mathematics item review committee. He served on the national mathematics advisory panel from 2006 through 2008.

His main effort in recent years has been directed at the writing of several textbooks for professional development of K-12 mathematics teachers, as well as providing summer professional development institutes for elementary and middle school teachers within the U.S. and abroad. Okay, let's give Dr. Wu a warm Pennsylvania welcome.

**HUNG-HSI WU:** Thank you. So I'm sorry to keep you waiting. You'll witness part of the drama. What happened was that I brought my computer here and I was downloading from the drive -- okay? Okay. All right, can you hear me still? Thank you. So anyway, on the way through the layover in Chicago, I turn on my computer, did not turn on. Oh?

**MAN:** I'm adjusting.

**HUNG-HSI WU:** Oh, I see, okay. So I load computer and fortunately, my son helped me to put all my files on Google Drive so I was confident. I come in and with a tech person around, download from Google Drive and good as new in five minutes, right? I've been working on this for an hour because, for some strange reason, one of the files could not be downloaded. Cannot be -- cannot be downloaded. Okay? Yeah, so but finally we had, well --

**MAN:** Keep talking so I can set it.

**HUNG-HSI WU:** Okay. I'll just -- so I mean, so finally this gentlemen helped. It's a long story. We finally had to do something [inaudible] and this gentlemen helped me. Anyway, so 45 megabytes, I think to download something, to download this.

So what you have actually is not -- I could have done that differently. Yeah, this is not the version, the final version. This was one of the drafts that happened to be downloadable. The other thing somehow was not downloadable. So I'll make some changes along the way, verbally at least. And so we'll have a go at it.

So I'll try to break somewhere around 11 o'clock so we don't -- two hours is too long for anyone, especially if I -- so I have no graphics, no pictures, no color, you know? Not the kind of thing you're used to, and so two hours of that would probably wear you down. So, but I'll break, yeah? All right, so I'll break just to give you water.

So we're going to talk about the implementing the common core mathematics standards, as they call it CCSSM, or that's what I call it. Common core state standards in mathematics. In the last 20 years, how many sets of standards have you gone through? Right. So is this one of those now?

This is the attitude taken by most publishers, by the way. They're not going to make changes in their books, by the way. You notice that too. They're not going to do it because they have seen -- they have seen so many versions -- so many sets of standards go through their office. And they have -- they do with not having put anything. Why should they do anything different now? So they're going to wait and see until this dies down too, and then they don't have to do anything, right?

So is this one of those? And I can assure that at least this is not one of those. This is different. So the -- so I should say something about the CCSM -- CCSSM versus TSM. What does it mean? I'll tell you what TSM means in a minute.

This is a general background about in what way the common core standards are different from all other sets of standards in our country. This is one of the Midwestern states, one of the public administrators that, well, we don't have to do anything because, after all, our organization has been doing so much, and all the national organizations have been doing so much. I'm sure we're prepared.

And I don't like to do this, but I think it is a fact that if this is the attitude you take, that, well, you know, we're ready, then I think you might find yourself in some kind of trouble. Because this set of standards makes rather unusual demands on -- not on any new gadgets and new gimmicks, new nothing, but on the mathematics. For the first time, this is a set of standards that concentrates on the mathematics in school mathematics. So that change, I don't think you can take it lightly.

The standards that you have lived through, most of them is another way of saying that before fractions, we're first taught in grade four and now it's grade five, or maybe multiplication was done in grade two, or whole numbers, and now it's grade three, that kind of thing. Shuffling, reshuffling of the existing topics. But common core doesn't do that. And the changes is not in, so to speak, rearranging the deck chairs on the Titanic. It's a matter of changing the substance of the mathematics.

So this is where we stand. Why do -- why does common core -- why does the common core standards have to make that kind of change? Well, this is where the TSM comes in. All the other standards take for granted that one thing remains the same, namely the mathematics. We know that doesn't change. So to improve means you are rearranging the topics.

I think it resonates with you. But the problem is that the mathematics, which is the mathematics, as you know, that is embedded in the textbooks, that part is not so good. Give the two explanations, in what way is the mathematics in the existing textbooks is not so good, all right?

So let's look at, for example, something as simple as teaching of adding fractions. Now the two ways to approach it, one is to say, well, children have no conceptual understanding. Therefore, we just add things by manipulators, by hands-on experiences. And so you draw a picture and say, one-half plus two-thirds, and then you draw a picture and say, oh yeah, this turns out to be -- I think it's five-sixths or maybe not. Yeah. Yeah, five-sixths. And so, you know, then you can see it.

But the other kind is, well, never mind understanding. I'll tell you how to do it. One-half plus two-thirds. And I'll mention this. The least common denominator, yes? And then you have a formula and then you do it. Never mind what it means. That's the way it is. You get the right answer. It's always correct, and so that's good.

Now the first way of drawing pictures, what's wrong with that? Because you draw pictures -- I give -- you draw pictures only, I'm going to ask you to add 125 over 77, plus 19 over 821. How are you going to draw pictures? And what does it mean anyway in that case?

The trouble with the second kind is that you keep on working and working without knowing what you're doing. After a while, can you go on? Children rebel, of course, when they say, I've been asked to do this time and time again. A little bit, it's okay, but if after five years I still do the same thing without knowing what it means, I'm not going to do it. Right? I mean, this is kind of a natural math rebellion, whatever you call it. That's what happens. And this kind of mathematics education really cannot go on. We have a mathematics education crisis. That's what it is.

So we want to achieve a little bit of real learning, so we have to change that. Another example is division of fractions. Now you all know you know invert and multiply, right? And so you either say it, first invert and multiply. And this is the great limerick, right? Ours is not to reason why, just invert and multiply. Or the other way is that, well, that doesn't work, right?

Let's have some real conceptual understanding, which translates into, again, picture drawing, simple things like one-fourth over one-half. You divide by one-half and you draw something. And then it's the four halves and you say, these is one-half. This is one-half of that, so this is one-half. Again, this is

-- you cannot go on with that kind of picture drawing ability. It's okay for lower grades. When you go to grade seven, grade 12, you know, you can't draw pictures. So neither would be -- would qualify as acceptable mathematics education. And I think the common core standards are cognizant of this fact, and they try to make a change.

So talking about why we cannot go on with this picture drawing or [inaudible] because division is a universal concept in mathematics. If you do it right, the division of whole numbers is exactly the same as the division of fractions, same as the division of rational numbers, which means rational numbers in mathematics means positive and negative fractions, not just positive fractions. Right? So if you like negative numbers, it's the same meaning. Divide real numbers, the same meaning. Divide complex numbers, the same meaning.

And that's what it means to do mathematics, which is that if something is good now -- what's that? Oh, I see. I'm hearing voices. If something is worth learning, it must be good enough forever. Otherwise, why do you bother spending the effort to learn it? You don't want to learn something and then five days from now, or even five months from now, you drop it. It's not worth it.

You want to teach things that -- which is actually true, it's not that difficult. You teach division, the division of whole numbers. Once you teach that, you can draw it and say, this is what we did before. We divide fractions the same way, divide negative fractions the same way, and so on all the way. That's how we'd like to see mathematics education proceed.

So we come to the TSM now. So the kind of mathematics that is in the textbooks, we call it TSM, textbook school mathematics. It's extremely flawed. It has been going on for a long time. And of course, for the two hours, coming to an hour and a half anyway, we'll be talking a lot about what -- take a look at what it's like and what ways we can improve on it, and so on.

So what the common core standards have tried to do, I think, is to eliminate TSM. It was not -- of course this is a lot of interpretation, but that's what it comes down to without any explicit statement. So the other standards pretty much accept TSM as is, and we're not going to do anything about it. But we can improve on it by reshuffling the topics. But common core standards, they say, let's face it head on. Let's do something about it, change it.

And this is why, when you see the common core standards, almost every -- one minor exception, the terms don't change. And like new math, some of you may know about new math. You know, all kinds of new terminologies, sets, the Venn diagrams, all of that thing, you don't see any of that in the common core standards. You see adding fractions, multiplying fractions, algebraic expressions, solving equations, prove Euclidian theorems, Euclidian geometric theorems. It’s all the same, almost all the same, with one exception.

Because the problem is not in the topics. The topics are perfectly okay. It’s the mathematics inherent in the topics that’s not so good. So keeping the topics fixed, we change things from inside. So this is exactly how this common core standards are different.

Now we have to fix the mathematics because if you don’t fix the mathematics, the non-learning is going to continue. And then you know exactly the computer adage: garbage in, garbage out. You’ll feed students the wrong things, the wrong things come out. When the wrong things come out, what do the educators in the education establishments say? They say, well, children don’t learn. Students not learning.

But children can learn. Students can learn. At least until I’m proven wrong, I have to assume that. But if you don’t teach them the right way, that’s why they don’t learn. So we have to implement the common core standards from this point of view. That is that we want -- we want to approach mathematics from the first point of view. Not fresh, from the correct points of view.

So I’ll give you examples. So right now, what we have, the problem is that our teachers have not been provided with this knowledge because our teachers have been taught TSM all the way when they go to college. Our universities don’t want to face up to that problem. Is anyone here who can tell me that in the four years in college, you have -- you came to some new understanding of the mathematics in K-12? Can anyone really say that? That’s exactly the problem. The institutions of higher learning really dropped the ball. Haven’t had anything for ages. And so now we’re left with this situation, so we have to do something to help our teachers.

So we need -- so the whole point of this presentation I’m going to give, more or less, is that we have to give non-TSM professional development. And I’ll give more examples. And what way we can -- what do I mean by improving on the professional development? And also talk about the general issues surrounding any attempt at doing this.

Now this is not you as policymakers, at least some of you here. This is not a kind of message like here because professional development is extremely expensive. Well, actually it’s not expensive at all if you do the kind that’s usually done. Get someone to come in, pat teachers on the back, and say, well, you know it, you know it, you know it. You'll be better teaching after this session. And they go back, expect, you know, one-day, two-day commitment, $3,000, $5,000 here and there.

But if you want our teachers to really learn something, this is not going to be one-day, two-day stuff. This is going to be long-term, sustained, and worse, you'll have to get people who actually are serious about mathematics, about content. Content-based professional development is a word -- a phrase that has been flaunted all over the place in the last five years because somehow content has become popular.

Unfortunately, many, many people -- many, many of the professionals themselves, they are brought up by TSM, don't forget, textbook school mathematics. And they don't teach you anything other than TSM. And at this juncture, that may not be what you need.

And now, of course you like to have -- the most important thing really is if you can get teachers better textbooks, they can all teach better. I don't think there's a single person who would disagree with this. Does anyone have any idea how to get publishers to give you better textbooks? The only reason I'm addressing this issue is that I don't know how to do it. I know how to do it, except that I'm not god.

So I'm not going to talk about it because no point in talking about something I have nothing to offer. They're not going to change. The only time they're going to change, I'll tell you this, and this is why non-TSM professional development is so important. You have to trust me if I say publishers will change their textbooks when the teachers themselves clamor for something better. And if we get our teachers to know better, they all say -- well, then they'll tell the publisher, you know, I'm not going to put up with this anymore. You give me -- don't give me this junk. They'll change very fast.

So let's address how we help our teachers. So without the textbooks, not a very good traditional professional development at the moment, we have a hard time. So if you accept that, then you know, there's the famous saying of Winston Churchill at the beginning of Second World War addressing his British citizens. Nothing but blood, toil, tears, and sweat. You may be too young to know this, but that happened actually, of course, to save human civilization in some sense. Because Britain for a while was fighting the Nazis alone.

Now of course, in our case -- well, they shed blood. There was a lot of blood in England with the bombing and everything. But we don't need to do that, so that part is just hyperbole, right? But there's no question about toil, tears, and sweat. Nothing short of that would get it done. So the question is, is it worth it? Now of course I'm going to convince you that it's worth it because it's not just saying children have to learn, but something much more is at stake.

Now so that's the general background about what the common core standards are trying to do. So let's talk about how the common core standards are usually presented to you. Not you, but to the whole country at the moment. So you all have heard of the practice standards, yes? This is the first ten pages of -- actually, first only three or four pages in the first -- among the first top ten pages of the document.

And this is becoming very famous. There are only eight practices. I'll list them in a minute. But people now think of the common core standards as saying, well, the only difference, the only way that common core standards are different from other standards is in having eight practice standards. So David Foster of the Silicon Valley Mathematics Initiative, my neighborhood, had an interview. And here is -- I'll let you read it yourself.

Now look, you read this, is there any toil, tears, and sweat here? Read the practice standards, practice it, students will learn. It's that easy, right? Students learn dramatically, right? Dramatic impact on student learning. And that, unfortunately, is the majority of you. That's what you want to hear everywhere, nationwide, statewide, and so on. That's what's going to -- that's what you'll ever get.

But I don't think that's good enough. So let's, first of all, remind you what the eight practice standards are. Make sense of problems and persevere in solving them. Now that is sort of -- you all know that. Read it abstractly and qualitatively. You know that too. Construct viable arguments and critique the reasoning of others, meaning students learn to talk out loud about mathematics. Model with mathematics. Now modeling is one of the emphases in the common core standards.

Use appropriate tools strategically, including how to use computers, calculators appropriately, how to use other spreadsheets and so on appropriately. Attend to precision. Well, again, you have that all the time, right? So look for and make use of structure. Well, that may be a little unusual, but structure, you mainly know what structure is. So that seems easy. Look for and express regularity in repeated reasoning. Well, so it's repeated, then you must be able to discern a pattern. So you figure that's okay too.

So we'll come back to these. So now I want to give you a dissenting view, which is that just looking at the practice standards, which is not normally what is happening normally, PD about common core, you have -- I think you have had probably even in Pennsylvania. In California, when this happens, people commonly talk about professional development addressing the common core standards. What it comes down to, talking about the eight practice standards, what they mean. How? By knowing that you're going to go home and do better.

What I want to claim is that, yes, they are important. But that's something you understand only after you have learned the mathematics. But you don't get it done just by talking about it to death. That's not going to work. So I'm going to convince you of that, right?

So let's -- in fact, so that's a little bit -- goes a step further. Let me see, do I have that? I think I have still that. See, I told you that this is a draft. I think maybe I didn't have that in the draft, but if not, I'll just tell you verbally why it's even harmful just to talk about the practice standards.

So unfortunately, I have to talk a little mathematics, which is not common, I think, in a general presentation of this kind. I have to actually talk about mathematics. Now I don't quite apologize for that. I won't go into great details, only for about just some general things. On the other hand, I think one of the problems that we have is that policymakers, when they come to decisions about mathematics education, generally tend to be divorced from mathematics. And that is a problem. Mathematics education is one of the things I will try to convince you that mathematics education is really different from other kinds of education. I'll tell you stories about that.

And so policymakers, when they come and make decisions, they -- even you don't have to teach mathematics. You don't have to know it inside out. But you should have a general feel about what mathematics is, what are some of the problems, some of the key issues. And so I'm going to go into that a little bit and give you my way of thinking about the issues.

And of course, you know, I'm not in administration at all, so if I have to tell you about how to do your job, I'll be wasting your time, so I won't do that. I'll talk about what I know, which is mathematics.

So the danger of people talking about practice standards and all the other things, all the things other than mathematics, is that it gives you this sort of view of mathematics education. Mathematics education, when all is said and done, comes down to problems within mathematics. And you'll have to make decisions based on some information about the mathematics.

So for example, why is mathematics education so different? In general terms, suppose you get a sub. You want to get a sub to talk about the Civil War tomorrow. Well, that person may not be a history major in college, but I bet you ten to one, sit down in front of the Internet, look up Wikipedia, look up all the associated articles, in three hours he'll find something to say the next day.

All right, try getting a sub to do fraction multiplication the next day. I'll give you three days. Do you think they'll get it done? Probably not. That is the difference. That's why you cannot look at mathematics education and say, oh, it's the same as reading, the same as English, same as -- it's different. So we have to face up to that.

So now I'll tell you another story. I was engaged in writing this volume. This you can look up at the library. So this volume addresses specifically the teachers of three kinds: science teachers, mathematics teachers, and reading teachers. So all of them how -- how to prepare those teachers and what to look out for, what the pitfalls are, and so on.

So because there were three groups, we would have usually just a short meeting, half an hour, doing the logistics. And they would disperse into three separate groups and discuss within our own discipline, right? I mean, math people, reading people, and then science people there. And we went on for a while. And one day, the organizer said, well, maybe there's something in common. Let's convene the whole group and have a long session.

So we met for one full afternoon. And so what happened was that we would go about our business as usual, except that we allowed the other two groups to listen in. So we went for about four hours, three groups. And then of course, the math people went at it and we went at it for a long time. Well, you know the math wars and everything, right? So there was a really serious debate.

So during tea time, the reading people came up to us. They were totally in awe. They said, we have no idea about the cognitive complexity of math education. Because we're taught things like fractions, [inaudible] third grader fraction. You know, simple minded things. And the third grade, you learn adding, subtracting. Well, that's -- you don't just talk about it. You have to actually come down to do it, right?

And then in seventh grade, and then you have to talk about adding not just fractions, but fractions with negative numerator, denominators. There's a rise in the cognitive level all the way through. But reading, reading is reading, right? So they just never came across it.

So that's one example of the difference between math education and other kinds of education. We get down to the nitty gritty and let's look at three simple examples, very simple. I'm sure you don't have to know any mathematics, but you remember from your schooldays certainly standard algorithm, right? Adding, subtracting, multiplying, dividing whole numbers. What could be simpler?

So on the calculator, you can get all that done in microseconds while you're learning it. Now that's an issue every teacher has to face. As a policymaker, you should ask [inaudible]. Well, in the normal way of -- normal way meaning TSM, in TSM, textbook school mathematics, these are four separate skills. You drill the students to learn those things because they're supposed to. No reason.

So the more open -- more what they call reform-minded people would say, well, there's no purpose of this. If they don't know how to learn the standard algorithm, which is still going on at this moment, let's just -- let them invent some algorithms. Let them figure out themselves. Draw pictures. That's another favorite, right? Get some understanding and that's good enough.

And then what happens when it comes to doing fractions, serious fractions? When all you know is drawing pictures, as I said, drawing pictures is great. It's great, it's important. But it has definite limitations. You cannot stop there.

So at the moment, what happens is in TSM, which is both, old or new, it doesn't matter, they just don't get it done. The problem is that we just don't look at it from a mathematical point of view. What is the purpose of learning standard algorithms? Well, I'll tell you what. Why do students learn it? Because learning algorithms is learning something very serious in mathematics. The whole purpose is that all four algorithms are not four separate things. They're only learning one thing. The one thing is that you want to teach the children that even if they can do single digit computations, which is simple, they can do anything.

Sort of like you go to gamble, put in $1 and $10 pops up. That's worth learning. This is the very fundamental idea of mathematics, which is to reduce the complex to the simple. Now if we cannot -- we cannot do mathematics without this basic modus operandi. This is how we go about doing it. And you better get kids started on real mathematics as soon as you can, right?

So you learn the standard algorithms, not learning a skill. You're learning a basic method of approaching mathematics. This is the crux of the matter. So now this -- I'll give you just a little bit, but for example, add a three-digit, two three digit -- three-digit numbers: 238, 451. Now see the payoff, right? Three simple facts. Two plus four, three plus five, eight plus one. You know what that is. Lo and behold, once you know that, you can add those three-digit numbers.

The three-digit -- that means you're counting, you're adding hundreds, right, by doing three simple one-digit computation. Think about that. What could be -- what could be better in this world? Something very complicated, we reduce it to something utterly trivial. And then, of course, you give the reason. So I won't want to press it too far, but it's right there. You just -- that's the reason. Place value and so on, right?

So at the end, you see this is what it comes down to, this is what it comes down to, this is what it comes down to. And then you -- those three things, that is why you get 689. Okay, so now why is this so good? That is the issue already confronted at any kind of elementary education. I don't care about this reform, non-reform, traditional thing.

The whole point is that addition is a very complicated affair. 231 -- what is it? 451 plus -- no, 238 plus 451 means what? Starting at 238, count how many steps? 451 steps. You should get a kid to try doing it in one morning. Try that. The whole point is that we can see that they'll get it wrong. They have to get it wrong. Nobody can count that much without making a mistake. They see it. They can't do it. They literally can't do it. And then you turn around and tell them, no, don't be dumb. Do this only and you'll get it done. Isn't it worth it? Yes, it is. They see it, right?

So that is the point. Now here we're talking about knowing precise definitions. How often have you seen definitions of addition emphasized in primary grades? They should be emphasizing it. The teachers should be. Because only when you know what the definition of addition is can we highlight why the addition algorithm is worth learning.

All right, now of course every teacher -- is there anybody, any teacher here, you think, well adding, the main thing is carrying. That's hard. We just go at it. That's wrong. Now I'm not saying that the fact is wrong, but the attitude is wrong because that is not the main -- that's not the real deal. The real deal is how you can reduce something that complicated to three simple one-digit computations.

When you understand that, then you know that's a little wrinkle in that particular approach. [inaudible]. Three plus eight is not a one-digit number, but I -- of course some of you noticed that, right? I purposely chose two numbers so that they'd add up to one-digit numbers, right? Eight plus one is nine. I cheated a little bit. I'm allowed to, right? But that's irrelevant, not important. You happen to be three plus eight, then you get 11, so that's a trivial little wrinkle, a little bump on the road, but you address that.

But if you have firmly in mind, fixed in your mind that it's a one-digit addition, that's important. That reduces the complexity. With that perspective, children will learn it much better. Trust me, try it.

So now let's try something else. And so that's addition [inaudible]. What about multiplication algorithm? So again my claim is that I want to multiply 43 plus 26. Sounds easy, right? I don't need to know too much. All I need to know is four one-digit combinations. Four times two, three times two, four times six, and three times six -- three times six. You know, those three -- those four facts, you can do it.

So I'm going to give you the details, right? So furthermore, I multiply 43 not by 26, but by 2. How do I do that? Well, you can see by the usual thing. You come down to this and that. But I know these two, therefore I know the end result, 86. So that's knowing single-digit multiplication means you can at least multiply any number by a single digit number.

So of course you do the same thing. Of course, 43 times 26, right? So I have to -- I pick two first and then I pick six. And the next thing, I do six, and of course it's identical. You do exactly the same thing, you get these two. And then you know those two, and therefore you get to 58.

So from single-digit multiplication, I know how to multiply any number by single-digit number. And the next step is, well, okay, knowing this, where to go? Well, it goes very simple. Again, you use place value and then now you lump the two and six together. It's 20 plus 6, therefore you multiply 42 -- 43, 43, you see, it comes down to this two, you see? Therefore, we did that before, right? You just add to get -- you get this.

And of course, this is a shorthand summary of those three pages. Or if you do it in class, it will take about four or five lessons. But it comes down to the same thing. All you have to do is multiply four single-digit numbers and you multiply these. And of course it extends. Give me two numbers, no matter how many digits, it comes down to the same principal.

Again, why is this so great? Well, because you have to know the meaning of multiplication. This is why mathematics, you don't know definitions and doesn't know where to go. You know the definition, the definition of 42 -- 43 times 26 is that you have to add 26 not once, not twice, not three times, but 43 times.

Again, would be good for your kids to make them do that addition. Make them try it. Maybe you don't want to say 43 times 26. Make it 43 times 12, let's say. Let them add 43 12 times, see how far they can go. Of course they make mistakes. You want them to make mistakes. And then you tell them it's actually simpler than that. Just do single-digit multiplications. So they can see with their own eyes the benefit of knowing mathematics.

Now this builds on -- I'll skip ahead, actually. We have trouble getting kids to memorize the multiplication table. Common phenomenon, right? There are high school kids, there are middle school kids who don't -- who cannot even multiply -- they don't know their times table. And you wonder, yes of course, of course there will be some who will never do it. That's -- we don't take that into account, outlier.

But in general, don't you think that you'll make kids understand learning the multiplication table is not a matter of someone ramming this down your throat, but you see with your own eyes that there's great benefit knowing this. Because once you know the multiplication table, one single digit multiplies single digits, you can multiply any two numbers no matter how many digits. That's a tremendous payoff.

You give something to look forward to. When you give them a purpose to learn something, you'll have a much better chance of making them learn -- of getting them to learn it. That's -- this is why you don't want TSM. TSM is a dead-end street. When you teach mathematics correctly, you can see all these byways that can improve student learning.

So what's the benefit? How do you -- and so this is coming back to recap, right? Why, in this particular case, why is it that if we can teach the standard algorithms correctly, you would think it would improve learning? Why is that? I'll give you two reasons. One is that, well, from the beginning, they don't say, oh, I am forced to learn four things. I don't know why I should, but I know I'll flunk the exam, right? That's not a good reason. Other than that. They'll just go through it day in, day in. I have to learn this, learn that. I don't know why.

But now you tell them, look, learn all this. This will show you one thing: how doing simple things gets you to do many complicated things. And many of them will respond better, at least better, right? And the second reason is that when you make believe that four things are complicated and serious, they have to learn it and don't ask why, they see, well, look, all you learn -- what you learn from this is one the one simple truth, namely how to do simple things. And from simple things, you could build up to very complicated things. Look at it that way. Children appreciate it. If you don't believe it, try it.

Children are very reasonable. They're open-minded. Or actually a third reason. Okay, sorry. That is the reason I would like to emphasize, which is that we are used to teaching mathematics because it's there. I would much prefer that you teach mathematics by telling them why they should learn something. Give them a purpose.

Mathematics, the whole of mathematics was created always with a purpose in mind. We should try to teach it in the same way. They learn standard algorithms not because they're supposed to, but because they're learning something very serious. Not really serious, but beneficial to them. Learn something simple, you get this tremendous dividend of being able to do complicated things. Well, they respond. They will respond.

So I mentioned that before, so I told you. Okay. So now here now, so we talked about how to do mathematics properly, right? So now then look back at the relevant practice standards. What practice standards among the eight are involved here? Well, at least four. To reason abstractly and quantitatively, attend to precision, look for and make use of structure, look for and express regularity and repeated reasoning. Okay, these are the four standards. I'll show you more precisely, right?

How did they come in? Well, when you say abstract reasoning, it's -- those words have been around for so long, you don't even know what they mean. But here you see a concrete example. The explanation of, for example, the division algorithm in multiplication algorithm, it depend -- you know, we have to go through the explanation and use the distributive law, use place value and so on. Well, that's abstract reasoning. They see it, but they are in fact reasoning used for particular -- for good purpose.

And then what about structure? What's structure? Structure is you go through four algorithms, you see with your own eyes that, in each case, it's a single-digit -- it's the single-digit computations that lay the foundation. No one [inaudible] will allow you to do any kind of multiplication, division, subtraction, addition. You have seen that, then you say, ah, it is true. In mathematics, you see this structure. There's always the same thing.

In four cases, you see the one thing running through all four of them, namely from simple to complex, simple to complex, simple to complex, simple to complex. Then you understand what they mean by structure. But you talk about structure, what does it mean? No substance means no understanding.

So the last thing is about the thing I already emphasized, which is the importance of knowing definitions, precise definitions, practice that [inaudible]. Attend to precision. Precision includes precise definitions.

Well, you know the precise definition of addition, precise definition of multiplication, you really begin to see it's worth it to learn how to add by this algorithm. It's worth it to learn how to multiply by this -- by the algorithm. So precise definition, I'll repeat it many more times, is not -- well, the average [inaudible] definition is. You know it, right? I'll say it for you. Is what? There's one more thing to memorize. That's all. Definition of -- but definition, definitions lie at the core of mathematics. Without definitions, basically there's no mathematics. And you've seen why it's important in this case.

So you make teachers go through this correct mathematical explanation and everything and correct understanding of the importance why you want to do it. Now you go look at the practice standards, then you appreciate it. You say, yeah, yeah, yeah, yeah, that's true. But not the other way around. You can discuss at length about what practice standards mean. How does it help you to learn to teach -- for example, how to teach the four algorithms better? Doesn't happen.

So this is a case of order. Learn the mathematics, then you see the practice standards are very important. But you want to drill teachers on the practice standards, that won't get them to learn the mathematics. And so it comes down to nothing. Okay?

Maybe I can go five more -- five more minutes? Yeah, because we started quite late. So let's go to another example. Equivalent fractions. So people at this end, tell me, doesn't it look familiar to you? It's in almost every textbook. Agreed? Well, you know from three -- two-thirds and eight-twelfths are the same, right? And it's because you multiply the top and bottom by the same number, four, and then what do you get?

Well, the explanation why is true. It is exactly -- what's in front of it? Multiply it, right? It's so reasonable. This, the same as multiply by one. One, you know four out of four is one, right? Multiply. And then you get this and you're good.

This is an old -- that's what TSM is all about. It gives you something that looks almost true. Now the reason is subtle. It is wrong for the reason that this explanation doesn't serve the purpose. And of course, that doesn't mean anything when I say it that way, right? The explanation is worse than the mistake itself, so let me explain better.

Why is it wrong? When do you do equivalent fractions? In the curriculum. Now it comes to a curricular -- curricular understanding of what you teach. It comes at the very beginning. As soon as you tell kids what a fraction is, almost the next breath you have to say, for fractions, something strange is happening. Namely, you can change the denominator and the numerator this way, multiply them by the same number. You still get the same fraction at the beginning.

So a kid learning fractions for the first time looks at this and he says, okay, that's what a fraction is, part of a whole piece of pizza and so on. And suddenly he sees -- he or she sees this. What is a -- what does multiplication mean to a kid at this point? Well [inaudible] whole numbers, yes. Multiplication of fractions. Why, after learning what a fraction is, you are -- you sort of ram this down their throats, right? Say, look at this, multiply. Okay, you multiplied.

Now this is so seductive. When you multiply like this, every kid's dream is what? Ignore fractions. Just look at the top, look at the bottom. That's -- you're living out their most elaborate dreams, right? So that's why TSM does this. It's so believable.

But you think about learning trajectory of a kid, they are uncertain, right? They don't even know what a fraction is. You say that is true, which they don't understand. Why? Because the fraction is true, of course. But you say that's true, it's because something else is true. They look at it, now the books make -- try to make believe that that's easy to understand, this part. Think about it. If you're a kid, you think that's easy to understand? Do you know what's going on? You don't. Of course, you don't dare say anything because the teacher is standing up there and he or she is the authority, and he said -- he or she said, so who are you going to say no, right? [inaudible].

You say, okay, I don't know enough. I'm not going to ask questions. What they say it is, let it go. There's nothing wrong with that, except you're trying to learn mathematics. Learning mathematics means what? You keep asking why. You keep questioning. You want to get the answer. Right? Off the bat, you taught yourself, I better not ask. I'll just take it passively.

That is the root of not learning. That's the root of math phobia. You just take what comes, don't ask questions. How far can you go in mathematics like that? Not far. That's not learning, right? An explanation has to be phrased in terms of things they already know so that they can draw on it and say, I know this. Oh, that leads to that? Okay, now I understand a new thing. Instead, you're telling them this new thing is true because something else that you don't know, but just come on. Just take it.

That's not mathematics education. That's not education at all. Now you say, well, it doesn't matter. But I say it does. So let's look at something -- some reasons. One is that if they start believing that's how you multiply, what do you think the next assumption is? That's the proper way to add. Because when you say, no, no, it doesn't happen, right, why not? They don't know multiplication. You told them that you just multiply the top, multiply the bottom.

Well, they don't know how to add either. Therefore, they have to say, well, if I don't know, it's do the obvious: top and bottom. That's not learning. That, you're planting the seeds of not learning.

The second thing is the psychological issue. They are uncertain. They are apprehensive. In that state, you tell them -- basically you tell them, come on, don't ask questions. When you don't know something, just go on. That's what it comes down to. These are long-term effects. Doesn't start there and die there. No, that's a cumulative thing of every time they come across something they don't understand, you just basically gloss over it and then they just say, well, that's what mathematics is. What if I don't understand? I just don't ask questions and go to the next issue, the next step. You can't learn mathematics that way.

So what's the relevant practice standard here? Students ought to -- they should not know, but they should be made aware of the fact that this is not correct. That's what -- part of what structure is. Structure is the way mathematics, mathematical facts, flow. And it's the natural course of development.

You define fractions. Yes, the next thing is you have to learn about equivalent fractions. That is the single dominant theme in all discussions of fractions. Once you know equivalent fractions, you can discuss multiplication of fractions, you can discuss addition of fractions. That's how the structure goes, like a building, first floor, second floor, third floor. There's no such thing as first floor, second floor, fifth floor, and then third floor. Doesn't happen. So kids should be aware, students should be aware of this particular aspect of mathematics, the structural stability, the way things go, naturally build up.

So if you say, well,  they can learn this if I discuss mathematical structure. So you talk about structure. All right, you want to -- teachers who understand common core standard by discussing the eight practice standards. Well, you see, some things cannot be drilled into your head.

I'll give you an example of democracy. You would just say, sit down, read the constitution, read the declaration of independence. Now you understand democracy. Does it happen? I know it doesn't happen. I've seen too many people from abroad, not to mention myself. Doesn't happen that way. I think that's a pretty good analogy. Why? One should not teach practice standards without first addressing mathematical substance.

Okay, so just to reiterate, when I say, you know, you have learned the mathematics, then you come back to appreciate it. And TSM, there's -- TSM is completely, I would say, oblivious of anything about structure. In everything, it's just on the spot of the -- on the spur of the moment. I want to overcome this obstacle. I don't care, I'll use anything under the sun so long as I get over this hump. Then I go to the next step. What structure? What are you talking about? If you keep this in mind, you go back and look at your textbook, you see it all the time.

Okay, so they're just, you know, talking, repeating the same thing. You cannot talk about structure abstractly and hope to change the teaching of mathematics. It's just not -- that's not happening. Okay, so now take a break, five, ten minutes, and come back for more examples.

Come back and I hope we can get started because you can imagine we started late, and then so we'll be short on time. We'll try -- we'll do the best we can. I might have to go a little faster, but I thought because we're discussing the mathematics, I had to slow down because otherwise there's no point even bringing up mathematics altogether, right?

So now the third topic is slightly more advanced because it's not exactly elementary. It's middle school, high school, more or less. So but it's so intuitive, so obvious, that I think you all have an understanding. So we'll talk about the slope of a line. So intuitively, it's quite clear.

You know, slope, it measures steepness of a line, how steep it is, right? So this is very steep and that's not steep at all. And of course, you go all the way down and it's flat, it's horizontal. So the question is, how do you measure the slope?

And in TSM, that's what you get. Now please look at this carefully. It says -- literally that's what it is. Let two points, P and Q, be chosen on the line L. And the rise and the run, which is the rise, it rises up, and the run. Now of course, I purposely choose to look at slopes like this, a line like this. The other kind, you just add a minus sign, so let's ignore that for the sake of our discussion.

So I ask you, what is wrong -- what could be wrong with that? When you look at this, anything wrong? Reasonable, right? Here's what could be wrong. Suppose you pick two other points, A and B. You pick A and B, then there's a rise, there's a run. So two rise and runs. Which one qualifies as the slope of the line?

Now this is never brought up, and of course never answered. If you don't bring it up, you don't answer it, right? The possibility is very real. What if they're not equal? Right? What if they're not equal? Then in that case, what is the slope of a line? The slope of a line is what? Is the rise and run associated with two magical points? Or is it any two points you do?

Now you say, well, of course this is academic. They get it and so on, right? Well, let's see. So first of all, actually it's okay. It doesn't matter which two points you pick. They're always the same, and the reason rise and run is independent of the choice of the points has to do with similar triangles. And this is the reason in the common core standards, similar triangles takes up about half of your year. And this is the reason why common core does not do algebra I. All right?

Now so when you do not make it explicit that you can take any two points to compare the slope, does it -- is it any problem? Well, you may think that -- well, TSM says, of course not a problem. TSM just [inaudible] gloss over it, and then proceed, make believe that nothing is the problem. And then if someone would need it, they would stealthily bring up two other points and use those two points to compute the slope. And they'd make believe that it's the same.

Now you know, it's very bad on the psychology of the student when things are sort of done under the table. And then they look at it and say, is this above board or is this something I'm supposed to know? Or is this, again, one of those things like that just lump it, don't ask questions? If they say it is, take it.

Well, that -- again, you know, you see the cumulative effect of this kind of education. So how do [inaudible] do it? Well, one, they say why describe, right? You slide over it and make believe that it doesn't matter. So then, of course, the price you pay is that because slope is the foundation on which all discussions of graph, of the linear equation rests. When you slide over the key point, therefore you have to slide over everything, which means that every fact about graphs of linear equations you take for granted. You just don't talk about it, including why the graph of a linear equation is a line.

So ask anyone, they cannot answer because the textbook never says that. You just say, you know, of course you know how it's done, right? You say, you plot a few points and you just look. Hey, doesn't it look straight? Oh yeah, that's a straight line. Well, obviously that's okay for sort of initial stage of convincing somebody. But learning mathematics? Learning mathematics is about painstaking verification of everything. It's reasoning. And we're not addressing reasoning, right?

Now the second -- okay, oh yeah, so when you slip over things like this, this is the latest in -- a year ago, NCSM newsletter [inaudible]. They did a survey of students and they said, what is the most difficult thing you find about the discussion of lines in algebra? The answer, how to find the slope of a line. Does this surprise you? You would think that's the most obvious, right? Rise over run, compute it.

And you may surmise, you may conjecture, but the difficulty is if it's never been explicit that any two points will do, kids have it fixed in their minds you compute the slope, you got to get your hands on those two magical points. If I don't, I won't make it. Therefore, you develop a line, compute a slope. Their unanswered question is, where are those two magical points? They're not there. They'll say, I don't know how to begin. That's my guess. I mean, you can guess your way, but that indicates what the problem is when you teach mathematics incorrectly.

So the other approach is you just assume that that's true. Even if you bring it out, you just say, well, that's true. Well, it raises many questions. We know mathematics is simply about reasoning. And this thing, it's not obvious. And then so when kids do that, they will say, well, why don't I take everything for granted and say don't bother, right? You do one, you do two, everything you can just take for granted. Don't ask for reasons. That's what mathematics education eventually degenerates into.

So you want the reasoning to be always there because mathematics is about reasoning. Without the reasoning, what happens? This phenomenon, I think every teacher -- I don't know if there are any middle school teachers -- every middle school teacher can tell you that's what they do. Before a test, there are four forms of the equation of a line. Which one is which? When to use it. Do I get it right or memorize it? Use the correct one on that particular occasion.

That's what happens when you don't have reasoning. If you know the reason, there's nothing to memorize, absolutely nothing. What I was given is just remember what -- you have gone through why linear equation, graph of linear equation allows you to say, oh yeah, you take a point, so on. So in the afternoon session, I'll address this a little bit.

So what happened is that if you -- well, you skip reasoning. You skip -- it's not just one thing, you see? It's once you do it once, you make people believe that, well, you could do it forever. That means other things in mathematics, no more reasoning.

Well, that's the wrong attitude. Mathematics is about reasoning. Well, you don't learn reasoning, that means you're not learning mathematics. And that's what common core tries to address, how to change this basic attitude. We're trying to explain everything.

So now coming back to, again, this is a fact, right? So now what -- which of the practice standards are involved? So certainly attend to precision, and reason abstractly and qualitatively. How's that? Well, let's see. Standard six says attend to precision, right? So what you want to say is that they should know the precise definition of slope. When we say precise, of course it means precise and correct. Precise and wrong, that's not what we want to see, right?

So they are supposed to know this definition really is not right. But if the teacher is not -- you just tell teachers, learn to reason abstractly. Learn to be precise. Generalities. And then you come down to specific things. So you think a teacher would learn that this definition of slope is just not precise and correct? Probably not.

And for example, we talked about textbook learning. I don't want to bring up [inaudible], but this is worth mentioning. I am not -- I've seen -- I haven't seen all the textbooks. I've seen a few. The few I have seen, slope is done way before similar triangles. In all the textbooks, so-called common core aligned textbooks, business as usual, TSM as usual. Just divorce the two things. Slope, one thing algebra, right? And similar triangles, that's geometry, separate. That's what happens.

So now talk about how number two standard -- practice standard number two comes in. That's called -- that is about abstract reasoning, yes? Yeah. So in this case, you can see that the reasoning involves the similar triangles that you can see why two arbitrary choice of points make no difference, right?

But that is very technical. You have to see it. A general discussion of abstract reasoning is not going to get it done. You don't have them learn that. So that's sort of -- I hope that gives you an idea why we want -- we want the mathematical substance in place first. Then you can look back at the practice standards and say, now I see, that's what you mean.

Okay, so let's reflect on the three examples a little bit. What do these examples teach us? They teach us that -- they teach us that, above all else, if we want to implement the common core standards, we have to help our teachers acquire the necessary contact knowledge. Do they need pedagogical skill? Yes, of course. You know everything and you cannot convey it to the other person, you may as well -- same as not knowing anything, right? As far as teaching is concerned.

On the other hand, all the teachers here, all the policymakers here, you know exactly the answer to the following question. Which of the two is the more important facing our teacher -- more important issue facing our teachers today? I think you all should say, I hope, the first one. Yes, there are people who are knowledgeable maybe, but they just cannot express themselves well. Yes, I see it all the time, especially in universities.

But by comparison, what do you think is the bigger problem? Well, without a doubt, content. Lack of content knowledge. When you don't have content knowledge, don't talk about pedagogical skills because you don't know nothing, and then you want to teach what? So we have to address the content issue.

So we have to help our teachers learn this. And unfortunately, that is hard to come by. Now so I'll give you three examples. If you want me to talk about this, I can do it for a week easily. Hire me, see what happens. Well, because I'm writing books for teachers in K-12, so I can give you any example you want. This is not the occasion. I just want to show you that it's not -- don't get the idea that, oh, three isolated examples, we can get over those examples. It's pervasive, all over the place. And this is why we have to do better.

But what's -- so I'm still trying to convince you that it's not true that common core standards is another one in a long line of standards. Wait, if you cannot cope with it, wait and in a few years it'll go away. It's not -- I hope it doesn't happen. And in any case, I think you will think it's a worthwhile cause to champion it and make it work. And that's the only reason I'm talking about this now.

And so you want to help teachers. Unfortunately, we don't have a very good foundation in PD, professional development. We think of PD -- well, I mentioned short-term, a few hours, twice a semester, whatever it is. Well, you have to have sustained, content-based PD.

Now the one problem I have is that I can tell you this and I'm not -- I'm not Arnie Duncan. I cannot say we're going to dish out federal grants and you apply and then I'll see if you're worth it. You know, I have no power. But that's something that they should. They might not have done it, but they should have done it, to tell everybody here is the time to rally around a worthy cause.

Well, the common -- you all have core standards. For the first time, we have a set of standards that begin to make some sense. They're not perfect. No, no, they're not perfect, but they're so far above everything we have ever had, we should start doing it now. We make the first step.

So we have to do PD, concentrate on it. The PD has to be addressing the content deficit in our teachers. Not their problem. They have a content deficit not because they didn't want to learn or they're just ignorant, none of that. Because we have a long tradition of not teaching our teachers the content knowledge teachers need to do their job. K-12, you know, textbook school mathematics. Universities, oh, they don't care very much. That's the problem.

So now, of course, I know what you're going to tell me. You're going to say, for me to talk like this to you, that's the wrong place, right? I should go back to my university and say, teach your teachers better. That's true. I know that's true. But that's a separate battle. It's a much more intricate battle. Academic politics being what it is, it's probably worse than the politics you face. So I'm avoiding the difficult issue and talking to you about the easier issue. All right? So in-service PD. Of course we should start with pre-service PD, right? We're trying. We're trying, but that's another discussion, all right?

So there are three main issues. How to get them the content knowledge, which is doing the right kind of PD. How to support them in their work. And if you have a good teacher with you, do you recognize it?

Now the time being what it is, I may have to be a little bit -- well, I think the rest, I think -- yeah, so that -- I'm repeating myself. So I think I'll just -- let's see if I can skip it. It's just that -- yeah, the rest is just saying no point doing practice standards before you teach teachers the mathematics they need to know, right? So let's go to doing PD, the difficulty of doing the right kind of PD.

So I mentioned this already. There's no good tradition in professional development. In fact, it's -- I think it's going to take place on the 11th in the Education Forum. What's it called? A forum or something like that. They're going to address this issue. I'm just quoting. Due to resources, professional development is still of the drive-by variety in most districts [inaudible]. That's the tradition we have, and obviously we should try to change it if we can.

And so even if it's a drive-by, it's okay if at least the drive-by variety deals with substance. But usually, the PD does what? Well, [inaudible] patting each other on the back and all this stuff. And then, you know, make them feel, yeah, yeah, yeah, you go back and you'll be better. That kind of -- that's not going to help.

And certainly lately, the PD means practice standards, practice standards, practice standards. That's not going to help either. We have to bear down and actually do content, the content that teachers are dying to get.

So yeah, so stay in content-based PD. So now I already told you there's no striking result for me to convince you and say our universities are trying to do better. I don't see that coming yet, but we'll fight, keep fighting. But just to say that getting university people, getting college people to come to do PD, not so fast.

I'll give you three reasons. One is that one must know college mathematics, which is what university people know. I mean, institutions of higher learning, they know college mathematics. That is very different from college mathematics. I'll tell you why in a minute.

Then second is that university and college mathematicians, they don't know about the school situation. They don't know school mathematics, the problems teachers have, textbooks, that kind of problem. And finally, you don't just grab anyone from an institution of higher learning and say, well, come help us do PD. If you do that, you're going to be in trouble. This is simply saying -- I'm considering myself in this in a later slide.

Like this, suppose you want to build a skyscraper. You have to hire an architect, no? You know you have to hire an architect, that's for sure. Do you just go out into the street any hire anyone, the first one you meet? Will that do? Of course not. So when you say you have to hire someone who's competent, I'm meaning literally that. Among the people out there, you have to get the best one.

So let's go into greater detail. You want every teacher to know as much mathematics as possible. That goes without saying. But we're not for the fancy mathematics, Gaussian curvature, rings and fields, complex analysis, nothing like that. The bread and butter issues of K-12 mathematics. And that is completely different from college mathematics because some of you have taken college mathematics courses, and you might reflect on the fact that whatever you learned in those courses, they are great except they bear very little on the mathematics you have to teach in schools. That's a fact. You have to have this recognition.

So therefore, you want someone who understands that a teacher, you just don't say a teacher knows this fancy stuff. Rather, you want the teacher to know the basics of the bread and butter issues. Know those and be able to explain them in language that children in K-12 can understand. That is not college mathematics.

So here is a pretty good example, one among, I don't know how many, 5,000 maybe. So college mathematics, what does it look like? Adding and multiplying fractions. So what happened is that college mathematics [inaudible], but deals with this issue in three lectures, or four at most. You have divide fractions. You say you have these two fractions, you call fractions. You say, how to add them, how do I multiply them? You just say, well, do it that way. That's how you add that, multiply.

This is the definition of the two operations, addition and multiplication. Well, of course I didn't say it in the slide, why do you do that? Because okay, from the point of view of abstract mathematics, you do it because you want to get a ring of fractions, the ring structure. But in school mathematics, what do you want to do? You start with the meaning of a fraction. By now in common core, they want to do a number line. And then you prove these things. Sorry, yeah. [inaudible]. Never mind.

So in school mathematics, those -- these are not definitions at all. This is particularly hard to prove. That, well, I don't have to talk about it, but that's also, well, problematic. These are things based on reasoning. You have [inaudible] starting points. College mathematics starts at a very high standpoint, and then you go down, so to speak.

But you talk about teaching innocent young kids, right? You start from the ground, you want to bring them up, show them the reason. It's just a different kind of mathematics. Both are valid. Nothing wrong with either. It's just that they're different.

So -- oh yeah, so we just talked about slope. Slope from the point of view of college mathematics is a non-issue because why? Because that's the definition. That gives you a linear equation. Okay, by the way, so a line -- right, now in school mathematics, a line is something that [inaudible]. Given two points, only one line joining them, that kind of a line. But in advanced mathematics, a line is, by definition, the graph form in an equation. So there's nothing to argue about.

What is the slope? The slope is, by definition, the division of these two numbers. Or minus sign is because what? It's a technical thing. Don't even worry about symmetry of triangles. No such thing. Don't worry about it. You see, so it's different.

So you get into university math or college mathematics, mathematician comes and helps you do PD, you have to make sure that that person knows this much, how different college mathematics is from school mathematics.

So number two about college people, university people. Do they know about the situation in schools? Well, I tell you that, you know, at least research the mathematicians. Try not to dabble in school mathematics for many, many, many reasons. One of them is that they're too busy with their own lives. They have lots of things to worry about, and this is not one of them.

And actually, the other reasons, well, the first one is what we're commenting on. Schools of education on campus have been divorced from departments of mathematics, in fact the department of anything, physics, whatever, so long that by now, they are really separate groups of people. There's no reason for either one to know the other. And mathematics education, as you know, is not one thing, right? It's not mathematics. And it's not education. It's both. It should be married.

That marriage has been dissolved long ago, and that's one of the intricacies that I told you about. Why not try to start with the university? Well, that's because the divorce has been in existence for so long, and you have to overcome that part.

But the other reason is that mathematicians generally tend to think that -- I have to see what I wrote. School education is a bottomless pit. You can do it forever and nothing happens. And that's discouraging, right? We go to do our own work, no matter how meager the result, at least there's some result. It's indisputable. You publish a paper and you lay your claim. But school mathematics education, what do you have? Well, I've been doing this for, well, more or less 15 years. What have I got to show? Not much. So that's not very encouraging.

So mathematicians, unfortunately, also have one thing against them, which is they believe in the intellectual trickledown theory, which is that you teach teachers advanced mathematics, then they know the elementary stuff. See, this is exactly the analog of the trickledown theory, right? Give you a lot of wealth, then you're going to do a lot of good.

That doesn't work. That doesn't work at all. Because if you know all that advanced mathematics, the only way you can let it trickle down to reform your thinking about elementary mathematics, that happens usually only with research mathematicians. You completely revamp your whole qualitative system, basically. That's very hard work. Doesn't happen often. You want teachers to know mathematics better? Teach it. That's the only way.

Okay, so the third one is how to recognize good teachers. No, I mean -- yeah, you have to have -- no, it's about why do you need competent mathematicians. Well, I already explained a little bit. You want someone to help you, you want the best person. Lot of people are sort of middle of the road and maybe incompetent, not there, right?

And it's just like doctors. You go to a new town, you want to get a doctor, do you just go -- walk into the first medical office? No, you ask you friend who is good, who is bad. It's that simple. So you should get someone who's really competent.

And so just to -- but just to explicitly address the issue, would you just say someone teachers at a university and is professor of mathematics, then he must know something? I don't want to comment on that except to use my own -- myself as an example. I was given tenure at UC Berkeley in 1968. I have time -- I have had time to reflect on why I've been doing education, school education, right, for a long time.

And I ask myself, suppose someone had asked me about school education around 1968? I was supposedly good enough, right? Would I have been any good? I think I would have been a total disaster because all I knew was the research level of things. I did not have a broad view. I was, you know, a research mathematician, had to be very -- know something deeply, but in a narrow range. But to do school mathematics well enough, you really have to have a broad view of mathematics, have seen enough. And that doesn't come cheap. So you have to choose carefully.

So when you go hire someone, get the best -- well, of course it's a vicious circle, but still you have to start somewhere, right? Get someone -- get some good advice, someone knowledgeable, some good -- and then do the best you can. Get the best person. And then if it doesn't work out, you ask yourself, how come it doesn't -- didn't work out? And then you try the next one and so on. There's no silver bullet, obviously. This issue is so complicated. Yeah.

So I'm repeating my -- of course I said it earlier. So the PD we want is not one thing, but both. First, the mathematics has to be good. But second, equally important fact, is that being good at mathematics is not good enough. College mathematics is perfectly good. We teach it all the time. But it has to be mathematics that's actually appropriate for K-12.

So set aside both conditions. Not going to be easy. But we still must try. So being as difficult as it is, I mean, I don't try to whitewash a difficult situation and try to make believe it's actually simple. But is there at least some model you can go on? So I propose one. So what I -- suppose a state, Pennsylvania or California, was to do something about this. What might be a strategy, usable strategy? I don't know how good, but at least maybe the only realistic one I know.

So you acknowledge the fact that we don't have enough knowledge, need a mathematician to help you. So let's start, try to get a few proven, reliable people. Let them train a small group of teachers at the top level. Do it for several years.

So when I say teachers, for elementary you cannot afford to teach all elementary teachers. That's pretty hopeless. So there's been a long discussion about using math specialists, but that's a separate discussion. [inaudible]. You can get into a long, detailed discussion. Meaning we want teachers who only teach mathematics in K-6.

So you put -- so put those and put the middle school math teachers and high school math teachers together. You get the best of those [inaudible] from diverse school districts, train them intensively three weeks every summer for a few years. You get what you call a new peers group.

So after a while, you'll be seeing that they are ready, they know enough. Send them back to their school districts and let them do PD for their own teachers. But to make sure they have a safeguard against any kind of accident, you also have a unit, a roving ambassador unit, go around the state and listen and watch. Make sure that the people you trust, you think will do a good job, really do a good job.

But that unit cannot be just mathematicians because they are probably still not as informed, well-informed, as -- about the school situation as a teacher. So the unit should have teachers and mathematicians together so that they can compare notes and confer to ensure that the dual role, the dual purposes are satisfied, namely the mathematics is good, that the mathematics used or taught is actually good enough for K-12, usable in K-12.

And then, of course, you hope that you do it for a long time, you generate enough people to be able to at least form a core group from which you can slowly develop a group of competent teachers.

So I just have to skip. So now we're talking about supporting teachers. District cannot just say, I get good teachers, and then ignore them. You need to give them support. They deserve it. And not only that, they need it.

So often districts don't do the right thing. And I can't tell you what not doing the right thing. All I can tell you is to cite three or four examples and show you how this is not the way to support teachers. But of course you'll prefer that I say this is -- I'll give you one, two, three, four, five, those that are ready to support your teachers, I can't do that. So I do the next best thing. I'll give you some examples how not to do it.

So you want to do no harm. That's the main thing. So I don't know if you heard of this. This is actually quite famous. At USD in the 90s, late 90s, they developed a pacing guide. You teach quadratic formula first and then square root later. I hear some laughing. So for those who happened not to be teaching those grades, you cannot even state the quadratic formula until you know what a square root is. But the pacing guide says teach the quadratic formula.

But of course, the reason is -- you all guessed this, right? Because with the state test, the state test comes at the end of May. And therefore, come hell or high water, you just get the quadratic formula out there first. That's doing harm. That's doing harm. That's not the way you want to support. You just completely pulled the rug under the feet of your algebra teachers, right? So you're trying not to make -- not make decisions of that nature.

So lots of people to blame for this, so I thought let's not discuss that. There's something like this, something so specific. And especially district-wide. They should have consult -- they should at least ask people who know a little bit of mathematics and that would have been avoided. Bring in mathematically knowledgeable people.

This is a [inaudible]. I mean, you cannot get -- so I, coming back to my -- I told you this presentation is about mathematics education is different from other kinds of education because the components of mathematics is so dominant. You cannot just say, oh, like any other subject. It is not like any other subject. Consult people who know some mathematics before you spread any kind of decision like this.

That's another example. So one state, I won't tell you what it is, some state was reading PD. And they put their attention -- then they focused attention on math. They said, okay, we teach successful. I don't know how successful. I'm not able to tell. They said, we did good in reading. Two exams, the same thing in math, that's good enough. So what did they do?

So they followed the reading recipe. The reading recipe is like this. Okay, I put there. One week, you convene the teachers. Pay them. Yeah, they even pay them. And then you just use the textbook, get some representative from the textbook's publisher, teach them from page one to page 852 each lesson, how you use the lesson. After five days, yes, you did -- they did go through the whole book for the semester.

And you see, this is what reading PD most of the time is. I think people can check me on this. How to use the reading strategies, all this -- I better not embarrass myself, but I mean, the standard thing about reading and [inaudible] that kind of thing. Well, if that's good enough for reading, we should just go through the motions. That's good enough for math. But that's not true.

They did this for three -- at least three years. And then what happens? No result. Of course no results. Because why? It's not that our teachers don't know how to use the textbooks. Furthermore, textbooks are no good. Secondly, they need the knowledge not how to use textbooks. That's doing harm. Right, this is not how you want to support your teachers. The money misspent.

So there, another kind of decision. These are state decisions. Not just one state, several. Talking about, well, we can adopt the common core by improving on the standards. And one way, a favorite way, is to say, well, we teach -- common core doesn't do algebra in grade eight. Let's do algebra in grade eight.

So that's what California actually did. I can give you the name of the state because it's on the web. It tells you the two options for grade eight. One, you go straight to common core grade eight, which is the normal progression. The other one is skip grade eight, go to the Algebra I in the Pathways. You know, appendix A, there’s an appendix B in the document. They go there.

Now that’s a childish thing because you cannot do that. The algebra in common core, high school algebra, depends on all the knowledge developed in grade eight. You are telling people to do something without any bridge, without any transition. That undercuts the effectiveness of your algebra teachers. What’s an algebra teacher going to do with people coming from grade seven and some in grade eight? What do they do about people in grade seven? They don’t know the material in grade eight. Now that’s not the way to do it.

Yeah, so I’m not saying the common core is perfect. By no means, but there are clearly some problems. But not doing algebra in grade eight is not one of them. It’s one of the strengths because the reason the common core doesn’t do algebra in grade eight is because they -- the common core recognizes what we call algebra in the traditional meaning of the word has been turned back lately because we are forcing students to learn about linear equations without supplying them with the necessary geometry background.

Therefore, they spend half the semester of -- half the year of grade eight on firming up the geometric background. And of course, when you do that, then you take half a semester away from algebra. So in grade eight -- in grade eight in common core, grade eight does not address simultaneous -- or no, it does, but doesn’t address polynomials, doesn’t address quadratic functions, quadratic equations. So that’s what it is.

Now textbooks. Lack of time, let me skip that. But I want to address -- how about I skip that too? Let me come to a conclusion. So when we are so focused on something, how to implement it, after a while, I might begin to give the impression that I’m playing a game. It’s in front of us; let’s see if we can get over this hurdle, right? It’s just a game, like any other thing, like a video game. Have to beat it.

I would be quite remiss in my obligations here if I gave you that impression. It’s not a game. What we’re talking about is something that affects the future of the country. We’re talking about the crisis in mathematics education, and we [inaudible]. We have this crisis only because -- you can phrase it any way you want, but it comes down to the fact that the mathematics inherently is not good enough, and that causes non-learning, TSM. What’s in there is just not good enough. I assure you that no reasoning [inaudible] and everything of that nature. They’re missing the whole point about, for example, teaching algorithms and all those things altogether.

And our kids are not learning. And we start blaming the kids, which is a standard phrase. But we shouldn't. We have to do better ourselves first. Then we can blame. Yeah, you do well enough. You can say -- now you can say, they're not learning because we're doing the best we can. But we have not done anywhere near the best we can.

So kids have stopped trying to make sense. When mathematics to them is a black box, that's a problem. So I'll quickly go through two examples of not making sense. Look at this. Simple enough problem, right? 1128 bus and you have 36 soldiers. Or no, 36 soldiers -- yeah, 1128 soldiers and each bus holds 36. So how many busses you need? So here is the statistics. Now of course, what you emphasize is that one-third of the students said 31 remainder 12, right? That's called not making sense.

Second one, this is more interesting. You might have seen. 1980, French research group. All right? You think that's fun, right? Look at the next one. [inaudible] not as Europe, not as France. And you will say wow. First and second graders, now if they have not been corrupted by TSM, you can't blame TSM. Oh yeah, you can because why? They get instructions from teachers. The teachers have been so corrupted already, and that's what happens. See?

So this is -- so you see, I didn't want to show you. So teachers have been immersed in TSM, and therefore they think the way they teach mathematics does not make sense, but to solve problems, look for key words. And so you go to google and google keywords and you get hundreds. And this is one of them I'll just show you. You see any of the phrases, you add. You see any of this, you divide, and so on. And this, you see, explains this. That explains that. Look for the words and never mind. It's H and this and that. Never mind, just the word. That's what you get, all right?

So we have -- this is a -- if you don't call that a crisis, I don't know what is. Many, many students doing that. So we have to help our teachers because they're not getting the help so far. So the reason this is on again is that we're talking about implementing the CCSSM. And the reason we want to do that is because we want to avert this national crisis.

So in the year 2007, some of you might know about this volume, Rising Above the Gathering Storm. This is on the web. You can download it for free. By the way, all these are from national academies. They're all for free. And so the say by 2050, there will be no leadership for our country, or before.

So what do they recommend? These are -- by the way, this is -- why is it so famous, I mean, nationwide? This volume was written by the CEOs of almost all the top tech countries and then also

Nobel Prize winners and so on. There's that sort of spectacular grouping of people. And they said we must increase America's talent pool by vastly improving K-12 science and mathematics education. No surprise.

In order to achieve this goal, to improve the talent pool, what do they recommend? Take a look. It's not a game. It's serious. We have to help our teachers to make them knowledgeable. So it's a -- you know, don't forget toil, tears, and sweat. You don't want to do that, you have a choice. But do you have a choice? Sometimes you don't. Look at Sandy. Sandy, you saw the incredible TV images. I don't know how they could. They are watching those things. Now they are still trying to regain some normalcy. It's impossible, almost impossible, but do they have a choice? No. Let's do it.

So that my only conclusion is, hard as it is, I'm not trying to gloss over things, it's hard. But let's do it. Thank you.