JARED CAMPBELL: So, good morning, again. My name is Jared Campbell. I'm an Educational Consultant at PaTTAN. And I have the pleasure of introducing to you this morning, Dr. Witzel. Bradley Witzel is an award winning teacher and professor who works as an Associate Professor, Assistant Department Chair and Education Program Coordinator at Winthrop University. Dr. Witzel has written several dozen research and practitioner articles as well as seven books including the best selling "RTI in Math" and the recently published "Building Number Sense through Corwin Press" and "Rigor For Students With Special Needs" through Taylor and Francis. He has developed and presented a dozen educational videos as well as delivered over 300 workshops and conference presentations. He was an elected member of the Smarter Balanced Assessment Consortium, Accessibility and Accommodations Work Group. And is a governing Board Member of the Southeast Regional Educational Laboratory or REL. A research hub funded by the Institute of Educational Sciences. Dr. Witzel currently serves as the editor of Focus on Inclusive Education through the Association of Childhood Education International and recently served as an author and panelist on the IES Practice Guide Assisting Students Struggling with Mathematics and as an invited Reviewer of the Final Report for the National Mathematics Advisory Panel. It is my pleasure to introduce to you today, Dr. Witzel.

DR. BRADLEY WITZEL: Yay, thank you. It was like buzz word, bingo. I appreciate that. You know, how many times did you say educational research in there but first, thank you. I appreciate you showing up. This is -- those of you who don't have power at home, I double appreciate it, or you're just trying to find electricity. I mean, maybe -- maybe that's where you came in, but this is a Web cast presentation, so not only am I asked to stay still as much as possible. Those of you who know me, that's going to be hard. My wife has already laughed at an email to me, "Ha, ha, good luck." Right? But we're going to make sure that if you ask a question, I'll be repeating it. So don't think I am -- I am a little strange but don't think I -- that's one of my weird ticks that I have. I'll just need to repeat things, so that the Web cast group can hear it as well. So hello at home. I guess you have electricity at home otherwise you -- it's -- this isn't working. The walls are going to have some video as well, so for your comments, I wanted you all to make sure that you get that as well. Give you a couple of things today, obviously we're talking about the Arithmetic, the Algebra gap which means, those of you who get to see right here, we're going to do some dock camera things. So I've got some live demo but I've also got some other things that we're going to see upfront. Your handout, those of you who elected the press print, 73 slides, right? So I just want to give you that warning on there. My goal is not to go through 73 slides. If you've worked with me in the past that is your information document. That's for you to go back, to porous, to see what's going on. All the slides are up here. Some I will spend time on, some I might say this is additional information on that topic. And that allows you to go back and see more of it, otherwise, I'll just be here babbling on about quoting research. When I think some of these is important to know the background but it's also very important to know what our role is going to be in that. **So audience, for just pacing piece,** I just need to find out who is who. Parents, raise your hand. That could be more than one role in your life by the way. So we got several parents and, you're parents of children in Algebra. Oh, my gosh, my nervous group.

That's where the questions will start coming at me. I understand. So I'll stand somewhere over there during that time. Let's see here, some different grade levels of teachers is here because I -- I -- I'm not going to ask you if you're Special Ed or Gen Ed because our role is mix. No matter who you think you are, your role is mixed. I got students who they thought they were low incidents, next thing you know they're a president of the university. So a lot of these has to do with identification measures. So in that regard, who's early childhood? My early childhood teachers, let's go P to Three -- Grade Three on that one. So just a couple. And please know, I said it on purpose because sometimes you hear Algebra and here comes the phobia like, "Oh, my gosh, I can't do this." But you guys are crazy enough to spend two hours hearing it. So God love you, and you deserve a hug or some of you are pushed. I don't know. But the point that I'm trying to make is early childhood has a part in this. This afternoon, there'll be some more clarifications. I'll be doing number sets. So we'll focus on early childhood in the afternoon but I want you to see some of that now, just touches of it. Okay. Where's my Grades Four to Six? Okay. Another small group in there. Let's go typical Middle School. Let's go Six through Eight. Some overlap. All right. Where's my High School teachers, then? Oh, majority are my High School teachers. Now, I'm going to split you in groups. How many -- how many of you work with High Schools and -- as an administrator? Okay. My gosh, your job, sir, are very intense. District Administration as well? Is there PaTTAN people as well here? We love our PaTTAN group. Okay. Who am I missing?

AUDIENCE MEMBER: Psychologist.

DR. BRADLEY WITZEL: Psychologist. I was thinking about the assessment groups. How many psyches are in the room? Okay. Only three that admitted? So -- I'm kidding. So we have three psychologists. Now, if you're raising your hand at home, it's a social issue. So I apologize to you. So what I want to do is we're going to get through as much as possible at the big points, the big picture and then we're going to start breaking it down into four different components. So the first one, it's a Math talk. So you know what we're going to do, solve these. Now, as you're solving these, some of these -- one of these will be an expression, one of these is an equation. As you work through these, I want to make sure that you're thinking about the processes of Mathematics that you're applying when you go through these problems. So I'll just give you about two minutes on it. Okay. Let's be honest, how many of you have done even trying? All right. There's a couple -- there's a couple of hands that didn't go up that should have gone up. They're like this right now. All right. How do you solve -- let's go to the equation solving for X in this left, 2x+5=18. Solving for X over here. What are the steps that you do? What's the first step?

AUDIENCE MEMBER: Subtraction.

AUDIENCE: Subtract.

DR. BRADLEY WITZEL: Typically we're thought to subtract first, right? Then what do we do?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Then we…

AUDIENCE MEMBER: Divide.

DR. BRADLEY WITZEL: …divide. Right now, if it was one X, you would still divide, right? Just clarification, yes. And then what's my answer? So what do you need to do to be able to solve for X in this simple equation? What Mathematics background must you be able to have? Subtraction, division…

AUDIENCE MEMBER: Order of operations.

DR. BRADLEY WITZEL: Order of operations. Right. And understanding of an equal sign or balance or however you want to call that today. What else? At least an introduction to the understanding of fractions. Okay. So we have to have some basic properties on this. I'm trying to dispel something to you. Okay. Let's go to the -- let's go to the right side. You're trying to work through and get this into some type of quadratic piece. What do you have to do? We have to do FOIL. Now, FOIL is what?

AUDIENCE: The distributive property.

DR. BRADLEY WITZEL: The distributive property. So make sure instead of saying FOIL because First Outer, Inner Last works for only these binomials. You start getting the more trinomials and more, more quadratics. It falls apart on students, so let's distributive properly. So if you have the basic understanding of the distributive property, we can work with that. What must you be able to do? So distributive property is one, what else?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Like terms, understanding what these terms are in this. I missed one. No.

AUDIENCE: Multiplication.

DR. BRADLEY WITZEL: Multiplication.

AUDIENCE: Exponents.

DR. BRADLEY WITZEL: Exponents. It's not even a scary exponential rules in this one. What else?

AUDIENCE: Negative numbers.

DR. BRADLEY WITZEL: Negative numbers. Oh, my gosh, the integers are coming. What else? Simple -- you said like terms. Well, that involves addition and subtraction. That's where those integers come in. And the reason why it might be is -- I'm trying to get those in front of that screen. It could be a negative Y here as well. So, in that regard, what grade levels of Math are we talking about to solve these two equations? A lot of fourth, some sixth, like the integers are sixth, the exponents might be sixth. What does that mean for the Arithmetic, the Algebra gap? This research study -- or this research whole line of studies, the Arithmetic, the Algebra gap. Where do the problems in Arithmetic come -- or excuse me, where do the problems in Algebra come from? You got it. So the reason why I make that point early is

because when you went through school, someone said, "When you get to Algebra, it gets much harder. Oh, you'll never pass Algebra with that approach." You get all these weird discussions. My daughter -- those -- some people in the room know my family, my daughters. My oldest daughter is in the Sixth Grade and she has been warned her entire life, "Watch out, when you get to Middle School, oh, so much harder." And for those who know my child -- my children, she averages 101 across all of her subjects right now in Middle School. She's bored to tears. She's actually asking for Fifth Grade to come back because it was harder. What she has found is that once she made Middle School, they're repeating the same things. The other day she came home with integers. She came home and then we're just chatting about this, she came home and she's [makes noise]. I go, "How are you, Laura?" "I'm fine. I'm fine." "How was school today?" "Well, I have no homework because it took me five minutes." She is upset. Please understand the take on this. "It took me five minutes." I said, "Well, isn't that good?" "You know, it would be good except that it's been three weeks now, and I'm still not learning anything new." "Well, what do you cover in school?" "We're doing integers, negative numbers." "Oh, okay. That's good though." She goes, "Dad, you taught that to me in the First Grade. I had to be six to know it's a number line, so everybody else around me do like, "Oh, I don't get it." And she goes, "Just go left." It's a number. All right. Go left. Really? She's frustrated to get three weeks of this. Good teacher, please understand this. Good teacher, gifted Ed students. Does that mean that she's a genius? No. What has made her prepare for this moment or be bored at this moment? Prior instruction. The Arithmetic **to** Algebra gap. We're going to cover some Algebra stuff, well, what happens now they're in Algebra, what do we need to do to help these kids understand? But we're going to also have to talk about what leads up to Algebra and how to change some approaches. Bless you. All right. Here we go. So the reason why we have to change some of these approaches, this study kind of sums up what a lot of our thoughts are for our early childhood. Seventy percent of the students, the lowest 10 percent in Kindergarten, 70 percent of their students, five years later are still in the lowest 10 percent, despite intervention, despite instruction. In other words, they came to school on the lower end and they stay on the lower end. And we're not talking about a small group. We're talking about a majority of the students. That's not good. So how I sum it up in my last book is, if they come in and you don't have specific interventions to prepare them long term, that's going to be the problem. Oftentimes we end up setting up our intervention, it's a Second Grade child who's struggling and the intervention is designed to get them through Second Grade. And so our success, we get them through Second Grade, we pat ourselves in the back, we go, "Yay, we made it." But the truth is, the goal is not to have the best seven-year-old in school. The goal is to get them through, not algebra. And I know here comes the scary news, Algebra II. The Alderman Studies in 2006, take this for students with disabilities as a Special Ed teacher. As I was a high school math Special Ed teacher. We used to say it was Algebra I, but we have watered down Algebra I so much at this point. The content is now Algebra II, so by the time this child gets off to college, this is Alderman Study for the US Department of Ed in 2006 looking through the California -- all the California community colleges. These students who could not get to keep the community colleges could not pass a college level Math. So in

other words, we think we're getting them through. We make it. He gets through Algebra. "Yeah, we got him." He graduates. He wants to get his job, so he goes to community college to learn to whatever that maybe. Maybe hopefully to be a **plumber**, and earn three times my salary, right? Right? And so he gets through. He's going to community college to get this Tech degree and then he can't get through because he can't do the Algebra at the college level. So what I'm going to try to do is when we discuss Algebra today, I hope I ramped it up a bit of what we're going to call Algebra. All right. Some of the things -- your background here. Some of the pieces. Why so many people hate Math? Those of us in PaTTAN, we've worked together now for 10 years. Ten years ago, I don't think we get this response. Why now? Why is Math at the forefront now?

AUDIENCE: Computer.

DR. BRADLEY WITZEL: Computer. The use of technology has increased, yes. Give me some other ones. The Keystones, the new assessments, the…

AUDIENCE: Globalization.

DR. BRADLEY WITZEL: The globalization. The world has become flat. So I have a friend of mine who -- no banks, no names. We're being videoed, right? But I have a friend of mine who is at a very large bank and he does statistics for a very large bank. And he didn't know what I was doing for a living, or just run a board for something in the community. And he goes, "You know what though, oh, that Algebra is really a waste of time." I go, "Okay. Why do you say that? What's your job?" So he explains his job to me. "I do statistics for this group." I go, "So you don't have to do that?" He goes, "No, I don't have to do any of that stuff." I said, "Hold on, hold on. Who's doing the Math?" He goes, "Oh, I don't have to." "Oh, no, no, no, I know that you're the manager. Who is doing the Math?" He goes, "Oh, we got a team in India who does all that." I said, "So you think we should just cut up all Math in our country because we're just going to farm it out to another country?" So here we are with this massive unemployment but you know who has jobs opening? Who has the highest amount -- number of jobs open in our country? Science, Tech, Engineering and Math, so we either have to do with complacency or we've got to make a change. So that's why we're going to talk about the Keystones. We're going to talk about the globalization. We've got some changes that need to be made. The expectations are one but to go to those expectations, we can't just increase expectations and say, "Now, I really get it." No, we have to change some of our instruction. So, here we go, some of the adult versions of this. I don't want to show the video. There are some funny ones. If you ever look up, "In The Definition of MPH", it's a great You Tube video. I don't know how these two would still be married. It's a husband and wife. And the husband asks a question to his wife, "We're traveling 80 miles per hour. If we do that for one hour, how many miles would we go?" And she can't figure it out. I mean she starts somewhat the rotation of the wheels and the size of the tires and trucks might go slower than us and -- you got me? I'm not -- I'm not -- I'm not blaming it -- I'm not claiming she's anything, smart or dumb. I'm telling you her background is that weak. So we have adult issues and for some of these, if you have ever worked in a restaurant, you're looking at this and going, I bet, right? We -

- if it comes with numbers, certain numbers, that's where we want to ground. Well, that's the point of what I'm getting at. Where are the details? Where are the -- where do we struggle? It's called the Arithmetic to Algebra gap. Students who seem to be doing okay in Arithmetic aren't always successful in Algebra. Tell me why?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: They can't transfer the knowledge. So let's start there. I love i-- thank you for the broad. So they can't transfer the knowledge. Where are the specifics they're not transferring? What does it take to solve Algebra equations? You're about to sum up $five million in research that we didn't have to spend. The generalization of -- okay, the Math rules, that there are Math rules and properties that must always be adhered to. What else? Fractions, multiplication. Listen, if you can't do -- if you can't multiply, your fractions are not going to work out easily. Division in fractions, we try to separate them and they belong together. Rational numbers in general come out. So when I ask this big question of why is there an Arithmetic to Algebra gap, there are going to be specific key components that we're going to have focus on today. Okay. But I want to make sure we clarified what the Algebra we're talking about is. I'm not talking about Algebra I. I'm talking about the formalized Algebra. If -- anybody ever get to read the National Math Panel Report? Great. They've got a -- they got a phonebook version of all the 6,000 research -- excuse me. I get few people upset. Sixteen thousand research studies, and then there's the final report, the executive summary which is about 70, 80 pages. They have 46 to 50 recommendations in there. And one of them is start focusing on the key components of Algebra which are different from what we sometimes think they are. They are much more complex, so I'll be trying to address those as well today. This is what it must be look like to parents. Moms, dads, this is what it must look like to parents, right? Here's our path to success. Well, you can do a little bit of this, a little bit of that but if you could go over here, then you're still okay, but -- right? There are multiple paths through success. What we've got to do is find the key components to get you there because we need bridges to go across this Algebra gap. So, first part, National Math Panel. I'm not going to spend a ton of time of it because you can do the background, you can do the homework. I just want to share some of the key findings for the National Math Panel. After that we're going to get into what is effective instruction. Those of you who know me and my research background, I can't help it. I want to get to what's called COA. I've got to get to it. I want to show you samples of it. I want to show you how to take a complex Math approach and break it down to simple -- to making sure that you understand that Math is actually has a simple approach in general. And then the last one, we'll talk about progressions. Math is a language. And if a different language is used in Fourth Grade, than is used in Seventh Grade, I will guarantee you, the child will enter Algebra with the knowledge of Algebra ready to go but he doesn't understand the language that are being discussed in formalized Algebra. So therefore, he's like, "I don't know. We didn't learn that last year," right? Then the [inaudible] "Well, those teachers don't know what they're doing." The truth is, they did learn it but you're using a different set of acronyms and associations that the students don't get. So we're going to talk about progressions and we're going to use the common core to show how a Second Grade

standard actually builds into an Algebra II standard. All right. So here we go. I like big diagrams and pretty pictures. And everybody likes the pyramids today, so I went to the pyramid but its not really. This is a four-sided pyramid, so triangular pyramid. You can fold them up. All sides should approach at least certain corners between curriculum and learner characteristics. I challenge, if you think you're not getting the results in your districts that you think you should be. Take a look at the learner characteristics of your district. Does it match the curriculum that you're using? I can give you IES studies. I can tell you the highest performing -- I think they're right now -- I think they move up to Fourth Grade but K-Four curriculums in our nation. I can tell you those but if the characteristics of your population don't match these studies, it doesn't help you. Research it. You got to be careful with the research. Research is very powerful but you got to match your students to the research that's out there. The other ones what I want to do today is instruction. I -- we'll be talking about instruction. I think instruction matters. I think it's one thing that kids are exposed to curriculum. I think it's much more important that someone teaches it to them. And as a parent of -- first, as a Special Ed teacher that was big, and then I was told by your children, you know, if they're gifted -- anybody have gifted children at home? If they're gifted, they don't need that kind of instruction and I'm here to tell you, oh, heavens they need twice that amount of instruction. In fact, they're begging for the instruction. Stop just present any things. Tell me where it's happening, why it's happening and what's going on next. They care about it. We've got to instill that across the whole set of students. So I'm going to do two things here, working memory and attention. We have actually geared down if you want a check a four-year old of how well they're going to do in school, you check their working memory and you check their attention spans, and you'll find out how well they'll probably do at the beginning of kindergarten. That's bad news. It should be much more complex than that but those are the key features. And I know there's Economics and all that but -- and the truth of the matter is, if they had these two things, they're almost ready to learn. So let's check your working memory. My working memory stinks. My wife will send me to the grocery store, "Oh, Brad, I got you a list, you will bring this list." "Okay, I got it." Here's the mistakes you'll make. On your way out, she'll say, "Oh, Brad, don't forget I need eggs, and I think we need bread. I think we're out of bread too. Add that to the list." "Got it." So, you know, and there's me, I get to the grocery store, I check up all of my stuff and I go, "Oh, she wanted two more things. E, it can't be elephants. E -- need yogurt?" So then I get to buy yogurt then there's, "B, I remember that bananas." So I grab yogurt and bananas and I come. She goes, "Oh, you got yogurt and bananas." "Yeah, yeah, you and the kids wanted that, right?" "Not really. But sure, Brad. Where's the eggs and the bread?" "Oh, my gosh, you know, Walmart was out of it. They were clear. Shelves were clear. There's no eggs left, there's no bread left. It's a Sunday night, that's just what happens." Well, you know, the truth is, she's gone to notice the milk was gone too. But -- then I am in South Carolina, so you know the milk was gone three days ahead of time. But that's a working memory issue. Because I've got one thing on my plate and I can't add to it. So let's check your working memory. Who here admits that they have a weak working memory? All right, all right. Now, I apologize -- I have to look -- oh, she's right in front. Emily, Emily, you are in the best position, I -- do you think I'm going to pick

on Emily? Quite the opposite. She is the one who I'm not going to pick on. Okay. Emily is the only one who will have to write, the rest of you are going to have to just remember, okay, so shake it off. Get ready. I'm going to give you three things to remember. Emily is going to take note them because I won't remember what I've just said. Seriously, I won't. Scripted lessons for me really help. Okay. So here we go ready? Forty-five, nineteen, fifty-eight. Okay. What I want you to do, take those numbers and write them down in the order -- on a piece of paper, write them down in the order that I presented them to you at. All right. Emily, read them out to us.

AUDIENCE MEMBER: Forty-five, nineteen, fifty-eight.

DR. BRADLEY WITZEL: So she said 45, 19, 58. How many people got them all correct? Keep them up. In the right order? Okay, good. Most of us could do that three step piece, quick three step memorization and easy things to memorize. I'd even make you adjusted with it. If I got real good working memory test. I'd make you do things with them. That's really messy. So let's get ready, let's get you up, let's work until five, okay? Here we go. You're right -- you're right with me, okay? All right. I got to remember that I'm saying five things. I'll be counting on my fingers to make sure I said five things. Ninety-four, seventy-three, two, forty-five, twelve. All right. Take those numbers, write them down in the order I presented them to you. And if you think you're confused, turn to your neighbor and giggle because they're doing the same thing. They just know slanch, right? How they'll sit up, lean forward, act interested, nod, and track, "Uh-huh, keeping up." Okay. All right, are you ready? Read them out, Emily.

AUDIENCE MEMBER: Ninety-four, seventy-three, two, forty-five, twelve.

DR. BRADLEY WITZEL: All right. How many people got all five of them correct in the correct order? Okay. Take a look at those guys, though, what is that? It's about a third of us, that's five steps. Now when you went through undergraduate, you read some work on the work of Piaget. And John Piaget said, "How many things in a row can you memorize?"

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Seven.

AUDIENCE: Plus or minus…

DR. BRADLEY WITZEL: Plus or minus three. When the truth of the matter is, even at five, we're struggling. But at that seven, we took it to heart. I mean, we took that seriously. You got your phone number set up this way. I've got steps in mathematics class, your textbook are set up this way, what's the problem with that? Even when it comes down to simple numbers, we can't remember -- in this room consistently but three. And if that child -- if a child has a difficult time with working memory, guys, I'm not even thinking three. How many would you trust in a row? One or two. Is that a question?

AUDIENCE MEMBER: Does it matter how you say them because if you had said the first one as a phone number…

DR. BRADLEY WITZEL: Oh.

AUDIENCE MEMBER: …we could have possibly remember.

DR. BRADLEY WITZEL: The chunking. So we're talking about chunking. So we got to talk about memorization pieces, so if you have a seven -- and here's the question then. Knowing this, how does that change your instruction? Knowing that we probably are stuck to one to two to -- maybe three, if we're really good, things at a time, how does that change your instruction?

AUDIENCE MEMBER: Keep the learning load light.

DR. BRADLEY WITZEL: Keep the learning load light. One, if we have to go high, we've got to start chunking things down the easy ways to remember them. That's where crossix come in, like words to memorize it. We're going to isolate the variable. Variable I stands for, S stands for, O stands for, right? What do you mean by keeping -- well, I don't want to put you on the spot.

AUDIENCE MEMBER: Sure.

DR. BRADLEY WITZEL: How -- what do you mean by keep the work load light?

AUDIENCE MEMBER: If we only teach one new thing at a time, you barely can get feedback from the students to make sure they know that one thing and then go to the next.

DR. BRADLEY WITZEL: Okay. I need you to teach my children, keep going, yes.

AUDIENCE MEMBER: Well, if I teach three things in one lesson oppose to teach one thing at a time.

DR. BRADLEY WITZEL: Now, do you all get it -- I'm going to repeat it because -- just for video. We may have to teach three things at a time in a lesson but we're going to practice them until mastery one at a time. So if I'm doing long division instead of babbling to the board about those McDonald's sold cheeseburgers, right, on Tuesday after dark or whatever that acronym is. And I'm dropping down, I'm bringing in the child trying to memorize and lock, step, now what moves were -- they don't understand what's going on, they just know what they're supposed to do. Stop. You can have a fully worked example, something to work off, but then you use what's called Incremental rehearsal. This Incremental Rehearsal allows me to, here's the first step. And on the whiteboards they may even do work with you holding up whiteboards back and forth, you're checking their increment. Did they get the first increment correct? Otherwise, you know what you end up doing? If you've ever seen this before, it's called Chicken Teaching. You might have been a chicken teacher once. You showed seven steps, you showed it like four or five times, you talked to them about it, they seemed to be interested, so you said, "Try the next 25 on your own." The rest of this class you are a chicken. And I'm so sorry for the camera because the rest of this class you're doing this. "Can you work the next?" "I can't even do this one." I'll be right there." And hands are above everywhere, and you just chicken teach from student to student to student, right? You start learning -- my back hurts, so you start learning the lunge technique. The lean-in. Ladies, you're

going real quick not to chicken teach. The older they were [makes noise] so you're on the lunge quickly and have a leaning in your hip. Some of you probably setup a table in the corner and said, "Line up if you got a question." Because you learn your instruction is can't -- you don't know how to break it down those steps. So the next step is, if you know about working memory, break things down into small steps. So I'm going to show an algebra side to that on a document. Can I grab my document? We -- oh, it's working. All right. Now, I got to zoom just because these are so small. Now, I stole this from Glencoe. The -- I love the teacher editions of certain things. So it says here -- increase? Okay. It says here, example, all right. Perform scale of multiplication and it's if J to the matrix of 4 negative 10, 14, 7 and find negative 2J. So, instead of that student trying to go through and memorize all the different steps that could be involved, I love the teacher versions because I just provide the potential steps out of order. Okay. And there's another step. So, the first thing I do is -- well, I probably need to set up the equation the way they said it, then I perform the scale of multiplication and then I can simplify it, right? Now, it's one thing to go through and they have to rearrange these steps, think about long steps, four or five, six or seven, it's pre-arrange for them. It helps them work it out. But what these allows you to do is get into the reasons behind them. They go, "Okay. Well, the first thing I did is I substituted, then I did the scale of multiplication and then I simplify it. So, by doing this it's already been preset for them of what's going to happen and when. Anybody have students who are in Algebra right now and they can't multiply yet? Okay. It's actually a large group in our country. So if you understand this side of an Algebra set in matrixes, you can also have it so everything has been prearranged -- I'm trying to find one of my handwritings that's not so evil. So they get the problem ahead of time. So that glare. Oh, let's go down. All right. It's a little better. But they can get the problem ahead of time and then the other steps -- the steps that follow it that they have to solve through -- hopefully all of my hands --oop, are already preset. So what's the -- what's the next thing you do? What's the next thing you do and what's the next thing you do. So in other words, a student who struggles in the multiplication has been set to what's going to happen next and why. Next and why. So those of you who are in the intervention side is right in the instruction I hope you understand it and break it down, but if I'm on the intervention side I need to step in and you can't say, "Well, I can't teach him this because he doesn't know what multiplication tables yet." You know, how many teachers I meet -- I work with dozens of districts in the United States and the major complaint of Algebra teachers is if they can't do the prior learning steps then I can't teach them Algebra behind it. And I keep going, "No, you find a way to build around the error, you're not going to be able to track through that error, you got to build around it." So my accommodation I've preset here is multiplication has been performed for them. They have to know the order of operations in order to get this correct. Fair enough? Just like they might not have memorized all the rules for scale of multiplication so I give the rules preset. They've got to put the order correct. And remember a lot of this has been done in our book all ready for. You blow it up on a piece of paper and cut them out. And let them rearrange it group by group. All right. Anybody know how to do that in their classes, though? Couple of hands up. Now, you can't phone a friend that was an easy question. I know. Stress with Math. Next, we're going to

talk about inattention and the inability of focus on task, but again we have to set our classrooms up to deal with that inattention. There are some other ones here I wanted to put on the National Math Panels point to this. They weren't talking about students with disabilities. Okay. I didn't talk about students with disabilities. I talked that intervention. What I'm making sure that we understand is this is a suicidal issue at this point. This is not something specific to those identified. It might be a husband thing too I think, but that's a totally different question. Low achievers, what kind of instruction do they learn the best with. National Math Panels identified it. Systematic instruction, that's explicit instruction and I have it two sides after this, but Anita Archer is coming next year and I got to tell you, I love Anita. Those of you who know Anita Archer, I love Dr. Archer, she's a good friend. I'll show you some of her ideas, but the truth of the matter is no one does it better than Anita herself so I'm going to share those with you and make sure we understand explicit instruction. They have modeled problems before them. Students need modeled problems fully worked examples. And I know she came from Carnegie Mellon. If students do not have fully worked examples they do -- they don't know how to go back and retrace their steps. Another one is in here. That's the example. The COA came out and then stop letting kids be like passive learners, working with students with -- students on the spectrum. A lot of times they go, "Well, you know, he's thinking it through." Now, even if he has to quietly do it, you've got to have the kid be active. Here's the difference between an adult version and then modeling for children. When I modeled with adults if you are connecting this to your classes, I don't know what you're doing here. If you're not connected to your -- to a child, I don't know what you're doing. But to a child who has nothing to connected to I've got to actively enforcing them to connect it, right? All right. So next piece, let's go to effective instruction components here. By Anita Archer work like I said I got two sides for it. You're going to see that Anita Archer took the exact same information to research and gives you 16 things that stand out for explicit instruction. When I work in one district we actually just check off these things for high school math teachers. This is I'm going to give you some background I won't tell you where, it's in the West Coast of the United States but we went out to the school three years ago, guys, they had very little things going on. They couldn't even get certified teachers. It's very rural and so we worked on them on some of these practices. So give you populations here 76 and 8 tenths are -- it was 93% is free and reduced lunch, just over 76% were Hispanic, Spanish speaking first and the unemployment rate in the town about 20%. So immediately a lot of us go to the, "Oh, I bet their scores were low." Well, within two years we took them from being near the bottom on their state, so they were just ranked. And I'm not loving US News, World Report, I don't love their ranking system. But their high school -- the one high school in town just got seventh highest ranked highest rate high school in the state just by going through explicit distraction and forcing it of step by step making sure the process is there, breaking it down for the students and then really examining some of the basics. So again, no magic, right? We're all looking for the magic here. No magic here, just going through some of the basics here. All right. So let's go to the some of the tricks. When you were told -- what's this fraction first? What's the fraction? I know some people are smiling who know me. What's the fraction?

AUDIENCE: Four and two-fifths.

DR. BRADLEY WITZEL: Four and two-fifths. Okay. You said four and two-fifths and I'm happy. All right. Because and means decimal or and addition. In this case it's going to mean both. But what do we do to solve this problem from a mix to an improper? What do we do?

AUDIENCE MEMBER: Multiply five.

DR. BRADLEY WITZEL: Multiply five times four.

AUDIENCE MEMBER: You have to add…

DR. BRADLEY WITZEL: Add the two. And then it becomes 22, and then what do I do? The five just moves right over -- you just move it over. What a magic, right? Five times four, add two, move it over, and guys I remember this in one of our upcoming books on The Arithmetic-to-Algebra Gap through Council for Exceptional Children. We actually have arrows and symbols and all the things we used to teach this. The problem, it's a trick. And not even a trick they can use outside of this grade level. When they get to Algebra I and Algebra II, the trick is not -- only is it not -- it won't work the same way, it's not -- the teacher is not going to allow that to happen. Because it's improper, it's -- well, it will be improper eventually, but it's inaccurate. So here's what I want to show. If it's four and two-fifths, it's really four plus two-fifths. I'm adding fractions with unlike denominators. Why do you multiply times five? Because you're trying to go twenty-fifths. So it's four over one plus two over five. Make sense? So I'm getting like denominators which caused it would be twenty-fifths plus two-fifths. Add the numerators, twenty-two, the denominator is a base. Do any of us who were in school long enough to use the teacher's base 2? Remember that, base six? All they're trying to do is set us up for multiple ideas in fractions. But it didn't help, we didn't catch the connection. But the truth is when you talk about fractions, you're moving the base. So you need the same base to be set up. Don't worry when we get to CRA, we'll talk about deviation of -- deviation, you like that? As myself coming out. We'll talk about division of fractions and trying to get the same base and it's not invert and multiply. There's other ways to do it that are sound academically. But in this case if you treat this like addition of fractions with unlike denominators, you're setting the child up. And I'll show why. But I want you to do one more. Here's that division of fractions. I'm going to tell you where it comes from, we'll do the intervention later. Two-thirds divided by one-fourth. What did our textbooks say to do? It's not for you to ask why, it's for you to invert and multiply or so I was told in school. I had one student named Ana, no last name just Ana and while I'm teaching and she said, "Why do we do this, Mr. Witzel?" And I went, "I don't know. What does your teacher guide say?" "Invert and multiply." "Why?" "I don't know." So went back and we investigated why. Two-thirds divided one-fourth by denominators of fraction. I would love that fraction to be one. If my fraction, the denominator of fraction is one, guys, then I don't have to worry about it. I can just go to the numerator. So how do I make this denominator fraction become one? Multiply it by its reciprocal. So it's four times one-fourth. Four, one-fourth says one. But if I do it down here I have to do it up top because of the identity rule, so I can multiply this same fraction division problem by one, four divided four. So the bottom is one, what happened in the top? Invert and multiply. Invert and multiply is not inaccurate, it's just -- it's just only half

of the problem. You're skipping half the work. So it wouldn't be easier just to memorize the stuff they're brought and do all these work, well, yeah, but I remember the goal is in Algebra. The goals at Algebra II, so a teacher of one of the districts I work with she said, "Thank you." Because we're doing a K-12 math institute somewhere institute, after a week and she sent this to me. She goes, "This is one of my student's work." So I just want to point this out. I had it darken over because it didn't scan well. But look, this is -- this is a mix fraction to an improper. That is a mix -- that's a complex mix fraction. It's 5 1/y, like three and one-half, it's 5 1/y. Another mix fraction. So I want to make those to an improper but look, to do that she makes them three [inaudible] additional fractions with unlike denominators. And both the numerator and the denominator. Now, she gets this but she needs to divide it out. Division of fractions now. You think she does an invert and multiple? No, because she wants to make sure they can do the identity principles. So down here, you'll see that kind of invert and multiply piece happens and the denominator and the numerator, so that this denominator fraction into being one. Some of you already intimidate just by y up there. Honestly, that's -- you were taught to be intimidated, you struggled at school, "Oh, he put y. But it's the same question as this and it's the same question as this. The difference? I put a y in there. And now, we call it Algebra. The Arithmetic-to-Algebra Gap means that the instruction that preceded Algebra was not designed to get Algebra. So later on I'll explain it and I'll give you the examples of where you can build this as early as kindergarten. But the truth of the matter is instruction up front pays off later on. This is why when you were getting people discussing, you know, teacher should be paid based on performance. You know I -- I'm not going to -- I'll make sure that I'll sound with it first but I want to make sure I clarify it. Yes, teacher should be paid based on performance but it shouldn't be grade level at a time. I don't think a Third Grade teacher I can truly measure their success at the end of Third Grade, right? So I was working with the governor of the state and he said, "You know, I like paid performance." And he you knew I was going to balk because we just wrote a paper to the states department about it that stopped their legislation on it. And he says well, "Let me finish, Brad. If I have a student who makes it to college and does really well and they came from a very tough background where no one else went to college, there should -- then we should celebrate that teacher." I said, "Who's the teacher?" The teacher would be the Twelfth Grade teacher, Eleventh Grade, Tenth Grade, Ninth Grade, Eighth, Seventh, Six, Five, Four, Three, Two, One, Kindergarten. If you went to a preschool that was paid for by the state, we need to pay these preschools. And if the parents went above and beyond, we need to -- we need to give those feedbacks to that parent." So don't think that they're insane people who are running this. What they're trying to do is change instruction, but when you get down to it, most people who run the laws, they knew exactly what's happening. Because I can patch things up in a Fifth Grade class and make it look good in Fifth Grade but it completely sets him up for failure in the sixth. So as a dad, I thought, those new -- we'll do videos later on. This afternoon I'll show videos of my children, what I did to them in kindergarten where my wife would be like, "What are you doing?" And like I said now they're bore because they've learned it all. It's called Mile Wide Inch Deep Curriculum and we're trying to get beyond it. To the child who has been successfully taught the way

through. Who has working memories just like me, very poor with my reading issues that I have. Now she's succeeding. It was about the up front work. So again, I worked with a -- here's a middle school teacher. I had to type his, he's even worst in handwriting than me. He was trying to just show some simple expressions here in Algebraic Expressions of differences of squares. And the students couldn't learn the Zs, so what did he do? He raised all the Zs on the board and just a five. And then he showed what it ended up with and does it match what we did over here, differences of squares and, of course, the student afterward is like, "Oh, I get it." Instead of just saying here with Arithmetic-to-Algebra Gab, he showed his as Arithmetic and then shared where it goes in Algebra. That's core -- Tier 1 Core Instruction. All right. Other things through there, I want to make sure we push some of these because I'm going to -- I'm going to try to speed past, but the bottom line is Me-We-Two-You. You've heard of I Do It, We Do it, You Do It? I worked with the state department and -- we'll describe the state department to give them credit. Kansas State Department actually is pushing Me-We-Two-You. Me, it's my turn, eyes in the front, watch me, then it's We, try the next one step incremental rehearsal at a time and then turn to your neighbor and explain the process that you just did. So we've got to have a step of instruction that forces active learning. So, fair enough? Who here does something like, how many of you force active learning? And what did you do? Tell me some of your tricks that force active learning? All at once.

AUDIENCE MEMBER: Choral response.

DR. BRADLEY WITZEL: Choral response, which means it's one, two, three, say it and then everybody says the rule or whatever. What's the property? One, two, three and they say it. Choral response is -- many of us think that's early childhood, I got to tell you, it works great in college classes. Because they got all these rules to memorize and then like, "I don't understand." All right. All right. All right. Stop, eyes in the front, watch me. Then it's your turn, "What did I just say? Turn to your neighbor, explain it in a different way and then you practice it." There's different ways we can get this through." All right. So up speed, so my first hour, I covered the background. I want to make sure I share with you the interventions that come with this. It's called the Concrete to Representation, to Abstract Sequence of Instruction. Who's got a little background of CRA? Who's heard of CRA? Okay. So we got about six to seven people in here. So if I do something that's not good, you're going to give me some feedback and share what I need to work on. Now, this is videoed which means you'd be able to watch some of the examples back, but Florida has actually done, this is Central Florida and South Florida. Florida has videos across the State of Florida that shows some of the processes. They don't get real deep in the Algebra side but at least they showed the CRA process. So if you want some background or want to share with colleagues, that's where you can get some of the videos on it. US Department of Ed by the way used to say the same thing of those videos sequestration, we've actually lost the contract to have those video showed. All right. Manipulative objects. Let me give you a warning about to show manipulatives. Manipulative objects do not teach children. I just want to make sure we're on the same page here. I've had children for, you can give them blocks, Math will not come out. They will not be factoring a polynomial by the end of the day, right? I can give them really cool little things and maybe they'll get bigger stuff, no, they won't

get place value with it either. How do they get it? Someone teaches it to them. Math is a language and is be taught. Instruction matters. So I'm not going to just manipulatives at children. I love the idea when they say, "I just want them to be exposed." That means the outcome was exposure. But if anybody else says, "Well, I want them to be exposed in multiple manipulatives." Well, that means the behavioral objective is that exposure is what we want. That's not instruction, that's okay. But understand it, you're not going into a math piece of this. I've also got teachers, who they have bins and they tell children, "Choose the one that you -- that make most sense to you." Okay. Now, it took Copernicus decades to figure out some of these things. It took Newton, right? Decades to figure out some of these things. He's not going to pull out the right bin and it's going to be magic. If that's the case, he didn't need to go in there in the first place, okay? So, let's go some -- we got some concrete misconceptions through this. I'm trying to clean up my little board here. Better than clean up my room. Something about secondary teachers, secondary teachers had the lowest and most infrequent use of concrete. Think about why -- I don't say you don't -- think about why by high school teaches don't often use concrete? And then I'm going to get in to some fractions with you. All right. I think I need to zoom, okay. Now, the first part is when we're trying to show a fraction, the first thing one to do this is to have something like a fraction strip. I'm going to jump into fractions. We said it was a big deal. The one participant doing that times with fractions were a big deal, so we hate fractions. But if -- and if you want to make sure fractions are understood, right? You fold them for agility fraction strips. You know why fraction strips were being pushed for a while there? All right. This would be a half, this would a fourth. The point was not to have more fraction strips. The point of having these fraction strips -- you see -- see what I'm trying to do here. Well, it's because if I lay out this fraction strip, it builds to what? A number line where I could go through -- well, it's getting me there. I can get it through it and I can identify where one-fourth is. Let's get a darker marker. Nobody going. Suit me well. And so it's one-fourth, two-fourths, and then three-fourths, and four-fourths. Now, I can have one student shade it this way, I can have another one that's halves and they're going to have it identified as one-half. I can get them the eighths right where eighths would come out. But you're helping students see that incrementally fractions when that denominator gets bigger, what is -- what's happening to the fraction?

AUDIENCE MEMBER: Smaller.

DR. BRADLEY WITZEL: You got it. I'm looking at smaller increments of the fraction. The fraction is small if it's an eighth, a sixteenth is a smaller distance than an eighth. And I'm talking about distances now because I've still going positives. Okay. So, so far so good, right? That's core, that's Tier I. Tier I if you're doing concrete first, it's a fraction strip and it builds to a number line. Once they get this number line then they can start analyzing it. The next steps that we go through -- I didn't bring a napkin. Bad professor. Naughty professor forgot his napkin. So when we go through that's the first step. You might say, you know but fractions are bigger than that. You got that fractions are also division. Fractions are also proportional. So you might pull it out, pattern box at one point. And said this is one-half, this is two-halves, equal one full, right? It's another way of looking at fractions. We call it Partitive. And listen,

partitioning fractions is not a bad thing. Circle graphs. I'm actually great with circle graphs as well if you've seen circle graphs and I've got some people like the National Math panel said, "Be careful of using circle graphs." They weren't anti -- oh, you're awesome, thank you. They weren't anti circle graphs what they're trying to do is why is circle -- what's the point of doing the circle graph here? If these were string, I'll cover my circumference on it. What does it open up to be? A number line again. So I'm not anti-circle graphs, just that the circle graph is a prior step to understanding this number line. Fair enough? Okay. Now that's Core Instruction. And I'm going to spend some time on intervention now. But for Core Instruction we've got to get number lines, we've got to get fractions strips, we've got to get distance to be understood. I got a Fifth Grade teacher who walked down the hall, I turned the corner, there's a little arrow. And as I'm walking suddenly a zero shows up, and 30 feet later there's a one. Students were lining up, they get sticky pads and they have to write down to get a fraction during the day. They have to line up according to their fraction from zero to one. So they take their sticky pad and like, "Well, I've got one-half. You've got three-eighths." Which side are you on? In other words, they're trying to physically work it out on the ground with measuring tape as well. There's lots of ways to build this. But sometimes even despite good instruction the student goes through and they still struggle. So let's talk about what to do with those beauties. Now, my children at home, we actually do this to begin with right after. I never -- I don't think I even showed the initial -- let's do two-thirds again. I don't think I showed the initial work on number lines until we -- until we've got some of the proportions down. This is proportions. This is two-thirds plus one-third. And I'm just going to say positives. I haven't introduced negatives and integers yet. So two-thirds plus one-third, fair enough? Well, if I'm adding fractions with like denominators, what do I do? What's the first thing -- if you -- there were regular notation on your -- on your chalkboard or your white board, right? Or you're awesome on your flier. What do I do first? I check that the denominators match. Okay. What's -- I'm sorry, what's the operation? Oh, adding or subtracting, good. That means I need to check, do the denominators match? Yes, the base is the same. We're talking about thirds. I then have my students physically move the signs up because once the base is set I know the answer is in thirds. Two plus one, are these two numbers going in the same or opposite directions on the number line? Same, so guess what? Two, three, what's my answer? Three-thirds, positive. Now can I -- can I change that to make it look easier to handle? Sure. One group of three, one group of three. If you don't see that it's one, I love popsicle sticks. They're so cheap and easy, you stock them, so the student understands three-thirds is the same as one over one. So far easy, right? Now, CRA, everybody likes to think of concrete CRA, but now, we started out with two-thirds, I thought so, right? Two-thirds plus one-third. What do we just had the student do? They checked the symbols, they checked the denominators then what did they do? Move the symbols up and I added two plus one is equal to three and the answer was in thirds. I then grouped them by threes because I end up showing you that we had one group of three over one group of three. Does that make sense? Okay. Now, what sounds magical about this so far? Is that easy to follow? Who might that be helpful for? I heard in everyone, well, in United States

probably but when we're thinking of our -- of our students, who might this kind of basic line instruction be helpful for? I'm sorry, what?

AUDIENCE MEMBER: Visual learners.

DR. BRADLEY WITZEL: Visual, students who need visuals. Students who need -- or are still by the way counting or we're going -- I'd get -- I don't do Early Childhood yet. I have some students that are still counting on their fingers. Welcome to Algebra and he's like, right. And he's tapping his desk and that student who say he can't do Algebra, he can't even count. Do you see where I'm going? We can still do the basic fractions and I'll get to Algebra. I can at least introduce basic fractions. Now, that was like denominators, I know what you're thinking. You're not convinced yet, most teachers never. So I'll go through another -- okay. Well, then how else can we break this down? Now, I want to introduce negative numbers first so we are in the same page with negative numbers. If I have positive three plus two, what's the question I asked again? Are these two going in the same or opposite directions? If they're going in the same they get them together. Cardinality, that's this afternoon, three, four, five. Now, watch this. Are these two numbers going in the same or opposite directions? Same. So what do I do? Three, four, five. This is just a number line. All we're doing is connecting a physical object to a number line. Now, I started this, but what happens to the green opposite. This is where you got your algebraic pairs and they call them algebraic pairs, but the truth is it's subtraction. One minus one is zero, one minus one is zero. Don't call them zero pairs, it means nothing to the kid. Have the child go out and say, "What is one minus one?" And if the child will be going like this, "Zero?" Good. This is two, we have to describe this again, okay? Say not -- not joking. We're going to have to go back and explain this again. One minus one is zero, one minus one is zero. What is three minus two? Positive one. Now, that may look like, well, what's the big deal there, Brad? But now take a look. Opposite directions in a number line, negative one plus one, zero. Negative one plus one, zero. Negative three plus two the answer is negative one. So, so far are we good? Can I get to the fractions, please? Okay. So let's set up negative -- oh, let's make it look like -- oh, yeah, there we go. Negative two-thirds, I'm sick with thirds if you're asking me to start doing this with fourteenths, I think you lost the point. And God love your kids, I have pictures of that. All right. We're there like counting out fourteenths times sevenths and they don't understand that fourteenths match up so they're getting into -- what will that be? A hundred twenty-eighths or something like that. Oh, God love that poor kid. I think you hated me at the end. Negative two-thirds plus one-third, what I do in this case? I check the operation. I am adding or subtracting therefore I check the denominator and right now -- bless you. The denominator is the same. So what do I do? Signs come up, negative two plus one same or opposite direction? Opposites. So what do I do? Pull one out. Zero-thirds doesn't help me. My answer? Negative one-third. Is that nice and quick? Okay. You want to get a little more exciting, start talking about different denominators. Now, to do that we're going to have to understand that I had two-thirds in there. What is an equivalent fraction? So if I had two-thirds, what's an equivalent fraction of two-thirds? Four-sixth. Now, as an adult we want the answer instantly. Let's get to the child and understand this here. I've got a set of two and a set of three. So, in this case, another numerator,

another denominator. We're not adding anything, guys. We're actually multiplying. So you see another numerator, another denominator. Now, I have four-sixth. The reason why this is not core or Tier 1 is because it doesn't look like I'm breaking things down smaller. This has to follow Tier 1 instruction. So what's another equivalent fraction to two-thirds? Another numerator, another denominator, and now I've got six-ninths. Okay. I'll move that out. What I said was I had two-thirds and I want to start getting light or I want to start getting equivalent fractions. So, another numerator, another denominator, another numerator, another denominator. Now, I'm about to break down fractions into addition or I'm going to break down fractions into it. It's all it is, right? We're overdone with it. Oh, it's fractions. Oh, it's an evil word, oh, but he laughs and goes and always laughs, he goes, "At a conference, if you say the word fraction, get ready for people to walk out." Okay. You know, just in a minute, it could be a reading conference blah, blah, fraction, blah, blah, blah, blah, huh? What? What did you say? Oh, your biggest upsets, [inaudible] people walk out on you. All right. So in this case -- let's go through this, so I'm going to have -- I'm going to make it nice and easy for myself. I'm going to get an answer bigger than [inaudible] because I'm going to take a little leap of faith for time reasons. So I'm just -- let's get positives here. I'm getting negative if necessary, but I just want to show the different denominators in this case, okay? I have one-half plus two-thirds, so what do I need to do here? I check number -- first thing you do, you check adding or subtracting and then they go, wait, the denominators aren't the same so we got to stop. So let's get an equivalent fraction of the first one that has a denominator that's a smaller number. So we got another numerator, another denominator. Fourths and thirds now. So, I'll go back here. I got another numerator, another denominator. Beginning to understand this multiplication table, so at this point, he's going, oh, oh, I think I'm close. Good. If not? You'll just go on with the multiples. So, you're just re-teaching multiples. And you may say in this one, another numerator, another denominator, you go check again now. Sixth and sixth. Great. So three-sixths plus four-sixths. Now, the denominators are -- became the same base. What do I do? Signs go up, same direction, so it's you can choose largest addend or not, but you can go three, four, five, six, seven. My answer is seven-sixth. So far so good, right? Okay. So if you like the improper, leave it there. If you're one or your book says, "It's go to be a mixed number. Are you ready to get -- we're going to take that step now, okay? I said this is a base for a reason, which means that once you get to six, you have one. Just like for us, we have -- we have this based handbook. We write it as base one. That's why we get confused on our numeration, right? Because every number is way over one, but then we call it base 10 because we're trying to say when you get the 10, we get to change over that base. So here's six, the truth is -- this is my answer, six-sixth plus one-sixth, right? My answer is six-sixth plus one-sixth. We're used to calling that one and one-sixth. Now, is this an important lesson for pre-algebra because one day, if you're going to get -- I'm taking a conceptual leap because one day they're going to come across a fraction and it's going to be x squared plus three divided by three. And what's that child going to do? "It's only x squared, I got x squared." Oh, and you say, "That's not right." "Oh, I'm sorry. X squared plus one." And goes, "Yeah." So you corrected yourself with an error. Awesome. I think I'll drive behind you in traffic one day. So in this case,

what do we need to do? This is a precursor to having has this conversation then. These three owns the numerator. It's X squared divided by three plus three. Don't get excited. Divided by three. That's what we're trying to work towards. So far, fractions make some since? Okay. So what do you think about that approach to fractions? Is that different from what you've seen? Has anybody tried something like that similarly? Not yet? Yes. You have tried? How's the success with your students?

AUDIENCE: Well, I haven't gone this far. I actually started using the popsicle sticks to have them see the fraction.

DR. BRADLEY WITZEL: Great.

AUDIENCE: And then we work on making the equivalent fractions using the popsicle sticks, so now seeing you go further ahead…

DR. BRADLEY WITZEL: To the next step to it?

AUDIENCE: Yeah. Oh, I'm interested to see how those tie together.

DR. BRADLEY WITZEL: Okay.

AUDIENCE: So now we're moving on to adding fractions [inaudible] that.

DR. BRADLEY WITZEL: You got it. And again, this is a two-hour session. I -- if we get a chance, I want to show you solving simple equations because you're not going to have one -- it's not going to be one X plus five equals eight all of his life. It's going to be a third X plus five equals negative seven, right? It's going to be something more complex to that -- with that coefficient. So -- but this is the precursor step. So, so far so good because I want to -- I want to make sure we cover all bases. All right. Now, the research has been strong in this and it's been very consistent. So you're going to find out IES. Now, IES, Institute of Education Sciences is the research branch of the US Department of Ed. So an IES says that this is something they agree to, that means the research has been clear. Now, this one is what's called a moderate effect size, which in research is good. It means that we have a few number of studies that all conclude success. And it says minimal effect size, that means there's only a couple of studies that had concluded success. This is a brand new field of research we've found. I say brand new. Brand new as far as math goes. It's been around since the 1890s in literacy. It's called multi-structural language. So the effect size, we just ran an analysis. If you know Paul, and Karen, Elizabeth. I think Elizabeth is now -- where is she in Pittsburgh? She had Duquesne, I believe. But we're running through this right now. We're trying to get through the details, and again that's our moderate effect size. Yes, teaching them steps is important. That's still the number one. They still need to know math has steps, right? This just in [makes noise] Math has steps. Yes. Yes, it does and you have to teach those. And the other ones that we're finding, the next highest effect, oh, technology, get them excited about it, but it depends on what they do to the technology. That's a follow-up research setting. The next strategy was CRA. So I give you some of these ideas. I'll make sure that you see these. This comes right out the -- some of the

federal guidelines. CRA, what I made sure that -- I hope you saw this. If you did it concretely, I could draw a picture of it. Did anybody try to take notes with tally marks? If you took notes with tally marks, you did the picto representation side. CRA means that they would have concrete, and I would model concrete with them. Once they mastered it, Id' show the exact same steps with pictures of a concrete. That's when the numbers can get a little bigger, and then if they master that, the numbers can get bigger yet and I start modeling with them how to do the abstract steps to it. How does this help students then? Because I'm not watering down anything. If there's a negative number, it's in there. If there's a quadratic, it's in there. The only thing it does, gives them a physical manipulative to think through logical steps. Motor planning, and if you don't believe in motor planning for students, then you probably don't want to see how your house is built, okay? You don't want the -- honestly, you don't want the architect to build your home. You want them to draw the plans for your home. You want someone who's good with motor memory to build that home. If you understood what I'm talking about and you see these homes go up in three days, you understand how your home was built. My kids laughed that we have a meek mansion because our home looks big to us, but I got to tell you, it went up in three days because these guys went like pop, pop, pop, done, next. Motor memory is powerful. This is a way of attaching motor memory to a mathematical process. What do you think the data says? This is just -- these are just two simple studies. We had them in a while back. When we put -- this is sixth graders. We tested sixth graders on solving equations such as three X plus five equals seven X minus four. Sixth graders don't always do well with that stuff, just wanted you to know. So this was the students with LD and [inaudible] if you just did abstract notation and just show them step by step with a good explicit instruction, they went from zero to three correct out of twenty-seven on the -- on the final. And the test was actually geared towards Algebra II students. We really want to push them. But for CRA, it went from zero to over seven. The same districts who got this a couple of digits, but, yeah, that works for those children, but you see, I also teach gifted so I wouldn't be using this. So what do you think their Math director told me to do? If it works with those children, I don't see the magic. Do with all of them. So here's the next steps, stanines from traditional instruction to multi-sensory CRA like the lowest stainines went for about one, like from zero to about one, this one went up to about four. In the -- in the post-test here, look at the middle stanines, but I wanted you to know those kids we call high achievers, the average Algebra II student had a 13 and a half on this test. After four weeks of instruction only using CRA, we had students in the Sixth Grade at the higher stanines outperforming Algebra II students. So I'm telling you, it's good instruction. I heard a voice. Oh, no. Okay. All right. I'll give you some more examples to this. I tried showing you these fractions. Right here is the solving for three plus X -- excuse me. Three plus one X equals seven in the same steps by dividing by the -- by the cop divisions, portative division. But I also wanted you to see how this works for addition. If you -- if you're catching on to how these new place values standards come, which is my last piece here, place value is going to takeover and I'm thanking goodness for that. As a churchgoer, I'm only thanking goodness because place value has been around for a long time. We just haven't paid attention to it. I work with people like Mahesh Sharma. I work with a lot of my -- a lot of my colleagues come from Asia.

They use an abacus. I mean, they're flying through an abacus and I'm going through my old calculator. "Hold on." Pop, pop, pop, pop. They're talking about the numbers of tens, ones, thousands instantly and I'm trying to get, what? We don't call that. So here comes these new standards and they approach it slightly different. All right. So with my examples, my examples, tick, tick, five, six, seven, I got lots of -- lots of examples. See? I got data. I told you I got data. Data is good. Here's the division, eight divided two, which means it's two-halves plus two-halves plus two-halves plus two-halves, eight-thirds. Three-thirds plus three-thirds plus three-thirds, thirteen-fourths, four-fourths plus four-fourths plus four-fourths plus one-fourth. The point is that you're not watering down the math, you're re-explaining it to students. All right. But again, I want to show you examples so you can always go back and review the examples. These were actually my backups in case that sucker didn't work. All right. I want to show you where we're building from. Young children need to start -- oop. My last action. Math progressions. My last half hour and five minutes. I grew up military, so I'm five minutes mentally behind. Math progressions. I can't get these things to working. I'm working with Vermont State Department and we're trying to get the numbers to stop counting by one, right? I got one block, two block, three -- everybody ever play a real board game with a child where you have to roll dice, right? And he rolls a four. What does the child end up doing now? The first time ever? One, two, three, four. So from my early experiences, my own children made we want to -- I was a high school person, studying only high school level mathematics, going, I need to start looking into this early childhood thing. I thought we'd counted correctly as a kid. Let's count together one, two -- no, from zero, one. One is a distance. So what we're trying to do is get the new, and I can't get it to slide correctly, like I want it to slide that way, but you're trying to get students to understand that when you move that one in there, you're moving from zero to one, one to two, two to three, three to four, four to five. It's a building of a -- of a number because if you do this, fractions are impossible. We've got to help the student understand fractions are possible eventually, such that here comes the magnitude piece. There, you get start getting those -- they're called length-based models. I've heard them call it strip diagrams, but I don't think I can say that in middle school. So, we'll call them length-based diagrams, but you can use those length-based diagrams to show which one's bigger by how much. Once they get this -- oops, the next step would be where is five on the number line? Where is three? But you can't do those. You can't ask that question until they understand distance. And by the way, if this is going positive, middle school teachers, we need one that goes negative with length. All right. Sybilla Beckmann, one of my buddies, there is a new fraction guide out from the US Department of Ed. If you look up practice guide fractions, you will get all of this and Sybilla's work and see those work and they'll see you where some of these things work out with that distance. They'll give you how to get from a strip diagram, right? Again length-based diagram, I'm getting better back to a number line and they also show the -- some of the common error patterns that work for that zero again. So how does these strip diagrams, how does these length-based diagrams work with number lines? Well, look at this. Farmer has seven cows, she's got four times as many cows -- horses as cows. How many more horses does she have? So you set up the diagram for number of cows then you said four times as many,

understanding the four groups and the question was how many more. It helps them see the length-based approach to it. Anyway, it's called schema-based problem solving, and if you ever hear of a good talk on schema-based problem solving, go to it because the problem solving we have in our students, well, read it, good luck. It's a reading comprehension issue. This helps you with the math and the reading comprehension issue. All right. But my goal is trying to get past the fractions in this into something we have to do call place value now. So when I said -- I said my numbers differently, my daughters when -- because going through the -- working with the National Math Panel and then working with those who wrote the common core, I knew very early on that math needed to change in its conception. So I'm still going to count from zero to nine, but when I get to ten, it's one-ten, one ten one, one ten two, one ten three, one ten four, one ten five. The new standards actually push that, kindergarteners, right? The new standards are getting the students to count with different place value language than we used to. Get ready to teach mom and dad because I'm going to show you why they're doing it. It's not -- we're all trying to get better kindergarten. We're trying to get them up to Algebra and the scientific notation later on. So just take a look here. Twenty-sevenths, two-tenths and seven-ones, so they would write down two and the tenths, seven and the ones. If they have to =, and that's -- I'll save it for the afternoon. Use some things that are proportional. So it looks like it's ten times as big as the number that proceeds it, okay? That's the base 10 block stuff. But what you're trying to do is for this question, my daughter's -- one of my daughters is a stinker, the other one is a goodwill follower. Those of you who have multiple children, right? First one rules, second one, not following those rules. Make my own. Look, I don't want to be like that. So, here they come. Here comes my second one and she had a really bright teacher in Second Grade and the teacher said, "243. How many hundreds? What's the -- how many hundreds are in this number?" And everybody said two and she raised her hand and she goes, "That's incorrect," but Caroline, there are twos. Teacher's very blank. And then she kind of looks and goes, "What do you mean?" She goes, "There's more than two hundreds in there. There are two and forty-three hundredths hundreds. She goes, "Oh. Oh, that stinker's of you. We haven't covered decimals yet." And then she goes, "For everybody, but Caroline, how many tens?" And they all said, "There's four," and she goes, "Absolutely not. There are more than twenty-four tens in that number." How many ones in that number? Two-hundred forty-three ones. Placed under -- when you understand your place value, and it comes from counting, when you understand your place value, you're going to start getting these new -- these new approaches that we've got going on here. So in this case, you're going to start seeing these standards where they're talking about expanded notation, decomposition of number. Some of us have been doing this for a while, but maybe we haven't totally thought about where it comes from. This is why where if you start hearing and the GPS say it's 1.2 miles away, my kids in the backseat now go, "It's one and two-tenths. It's one and two-tenths. What's wrong with her? Can we get a smarter GPS?" All right? Can Siri get on the ball? Let's go. So, here's the Fourth Grade standard, 24 times 76? Give me the new numbers. Two-tenths, four-ones times seven-tenths, six-ones. Now you're going to see a lot of these are F -- are using that that's rectangle array or area models. So, the students are going to be able to solve it,

but we want them to be able to illustrate it using a rectangle array or an area model. This is what they mean. So anybody who are doing this? Anybody that work towards our ways? Good. We're all going to have to. This is what we're assessed on now. So take a look. I'm not going to call it 76, seven-tenths, six-ones, two-tenths, four-ones. So, hear my language through this. Seven-tenths times two-tenths? First things you heard, seven times two is fourteen. Ten times ten is hundred. The answer is 1400. Six-ones times two-tenths? Six times two is twelve, ones times tenths, the answer is in tenths. Same thing, seven-tenths times four, twenty eight tenths, and six times four is twenty four. Now, I know what you might be thinking now, my gosh, that's a lot of work, oh, by the way these are -- these products, once you get these, you add all them up. And I know that you're thinking, that sounds like it's much more work. Why would I do that in Fourth Grade? It's going to cause me more pain. No. Hold on. It's called a progression. You're not planning to be the best -- I don't want the best fourth grade as a dad. I don't want my daughter to come and be the best fourth grader in the world. That's not my goal. Here's what I want her to start building from, Fourth Grade also does the same multiplication approaches of two times three-fourths. Before we had tenths, now we're going to have fourths, remember the base. Two times three is six, the answer is in fourths. They're trying to set you up, and fractions are harder to see than decimals, but one more fraction, and then I want to just -- well get to interventions later on with these, because this is the core, this is Tier 1 now. So, same thing, two and one third times four and one half. Two times four is eight, four times one third, first things you hear, four times one is four, the answer is in thirds. Two times one half -- I don't like halves. Why can't they call them seconds? But anyway, I know they're not time, come on now. I think the Olympics would go crazy with that, right? He got -- he got second seconds, but anyway, and he got second place. But it'd be two times one half, two times one is two, the answer is in halves. One third times one half, one times one is one, halves times thirds is sixth, then you add up the answers. So, far is this -- is this okay? Remember, the goal is no longer to get Fifth Grade. It's to start moving on. Now, this is going to be easier for us to understand because we know this language. Seven and sixth tenths, two and four tenths. That has been shoved down our throats. We know it's called a ten to precision. We know that for saying point forward, we're getting it wrong. But we still do it colloquially and I do it all the time and get confused and my kids correct me because they weren't indoctrinated differently than I was into this number system. So seven times two, fourteen, right? You're used to two times six, carry the decimal, listen to what's happening. Two times six-tenths, two times six is twelve, the answer is in tenths. Twelve tenths is 1.2, right? That's the place value difference that our children have to start getting. Seven times four-tenths, seven times four is twenty eight, the answer is in tenths. Four-tenths times six-tenths, four times six is twenty four, tenths times tenths is -- say it to your hand, not your neighbor, hundredths, right? That's a lot of spittle. But then you answer it right, you add up the products here. Has anybody been starting that already to that level? Good. And you're going to start seeing more websites come out with it, and they're not US based. You'll start getting more influence because the Japanese math standards, which was so influential in our common core, you're going to see a lot of Japanese programs coming through. And listen, remember I survived the Japanese

game show, remember that? It didn't last but a season, but you get a 22, and he goes. Oh, [inaudible] he goes, two-tenths, two-ones, and he's doing it left to right. It is a left to right counting system. We don't do [inaudible] thirteen, fourteen. Ever have kids mess it up and do -- write down 41 instead of 14? He doesn't have dyscalculia. He's got bad teacher. What we need to do is reinvent the math -- the number system so he gets it. All right. You're going, "But I do that." I know. I know. Don't be mad. We're all in the same system. This is a cultural change. Okay. Well, where does this come from? What are all these models trying to stimulate? Cut a foil earlier, it's really what property? This is all the distributive properties, so get ready. 3x-1 times 4x plus 5. What's happening? 3x times 4x, first things you hear. Three times four is twelve, X times X is X squared. I could take out X and put in the zero and I have the exact same verbiage except the X. The point of this is that Algebra is not unique, it's nothing new. All Algebra is doing is explaining the rigor of the arithmetic they should have learned. Algebra is not supposed to be a barrier course. It's not supposed to be the gateway course. It was never -- that was Calculus, guys. Algebra is a pre-calc. And if you -- when you get a calculus, you know what you do? You move on to college Algebra. You get me? It's -- it doesn't go away. Algebra is not a course. It's a thought. So in that thought process of this, we have got to change how we think about it. -1 times 4x, -1 times 4 = -4. The answer is in C. Does this all make sense? Okay. The reasons why I'm trying to share you these with polynomials is and I'm going to go back, each one of these steps build year after year. The goal has always been to get through Algebra II, not to try to do better in Fourth Grade, or Third Grade. And I know this upsets sometimes as a daddy, "Why is my kids not doing perfect in school?" Maybe the goal isn't this year. Maybe the goal is next year. So, I want to introduce -- I see we're heading to the floodgates. But I want to show where the interventions come from. I don't -- I don't have the time, but I want you to show National Training Network. The National Training Work, you can actually go and watch videos of how they do this concretely. And that's free of charge. And, of course, we'd like you to buy their textbooks, but I'm explaining to you that you can watch their professional development. Make no money off them. It's a good group and they've got real good demos with the teachers hand just like by Doc. Campbell showing this. That's first part. But I want to show to you where my students [inaudible] intervention [inaudible] went through. And I want to show what the array model is, where we started on this one. Just take a look at this fraction strips up here. I've got two-fourths time two-fifths. [inaudible] standing in the front, two -fourths times two fifths. These are basically --I threw it over here, fraction strips is all we're doing here. And we're trying to show how it works for multiplication. So, now I'm going to go, two times two is four, four times five is twentieths. So listen to how these works out. This young lady right -- wait, where is she? She's right here. This young lady right here with a learning disability comes [inaudible] since Sixth Grade. She couldn't do any of these fractions and I love -- her intervention [inaudible] is great, they end up showing her fraction strips. You can see the popsicles sticks are out there for other lessons. This is a place value teaching here. But they showed this here, and they're walking through it, and that young lady goes, "But Dr. Witzel, I want to show you what I think makes sense." And this teacher here goes, "Yeah. We're trying to figure it out." All they're doing is getting a

positive student to talk. A teacher would know this, but what she does is she takes these, she takes four cubes here, four pop cubes on top, she goes, "So right now it's four out of twenty, which is four twentieths. But what happens if I move this pop cube here and this pop cube here?" I said, "What do you mean?" "You told us early on in the group, because when you group and we break off into small teams for intervention, like group lesson, small teams. That way, the differentiation and accurate language up front. So, when she -- so she goes, and she goes, "You told us that division is equal parts, breaking something into small, but equal parts." She goes, "But also division can end up being the same -- the same answer if you divide it by one." I said, oh, "Well, tell me, what are you coming up with?" "Is four-twentieth the same as -- remember there's now a cube up top all these row here. Is four-twentieth the same as one-fifth? Because I've got one-fifth, one fifth, one fifth, one fifth. Is four-twentieth the same as one-fifth?" She just broke down the definition of division. And so of course I go, "Well, why does that make sense to you?" "Listen, I know -- we're used to, in special ed, going, "Yes, no, we want immediate feedback." She's already been giving herself feedback on it. I just want to work with her at this point. Why do you think that makes sense? She goes, "Was it right? I go, "Absolutely. I want to hear why". She goes, "Because I broke down the numerator which was four. I broke it into four equal parts, and then the twenty, twenty divided four is, and her little fingers are coming out, is it five?" I go, "I think you just defined division and fractions. It's called equivalent fractions. Good job, right?" Now, we can call simplification or renaming fractions, listen, equivalent fractions is really where it comes from. So, in this case, intervention using a concrete, you can do the exact same discussion. All right. So, I want to show you some of the decimal versions with you. In another word, you can actually grab just nice big graph paper and work it out. Click. So, then, Sharpei, here Sharpei, here yeah, okay, good boy. All right. So if I'm trying to do this and I'm going one and one-tenth times three-tenths, I go across here and I find five. I can only group by this. Here's one and one-tenth. This is three -tenths, so it's three, but it's three-tenths, so I got to go down to tenths. I black out tenths times tenths are hundreds. One and one-tenths, so it's one times three-tenths which would be thirty-hundredths, plus what? Wait, here comes another one here. What would it be? Thirty-hundredths plus, I don't -- I've went out of room here. An additional -- switch back. Good. An additional three hundredths because I've broken down one and three-tenths into one and three-tenths, so now I end up going by tenths times tenths and then it's three-tenths times six-tenths which is eighteen-hundredths. The reason why you end up showing this way is they get to see the hundredths. And by the way, if you like the concrete version what would you do instead? You take out based in blocks, I've got my hundred square. It's not a flat, right? Save it for England. It's a flat, right? These are longs and these are shorts, but the truth of the matter is, this becomes one, this becomes one-hundredth and this becomes one-tenth and allows me to do the computation over that way. Let me show you what I mean. Same thing, four and three-tenths, so, I grab those flats and longs, and I grab my four flats, one, two, three, four, it means four-ones, four-tenths times two-ones, four-tenths. This is the distributive property. That is all we're doing. So, what is four times two? Eight. Now, here's where we start getting tougher with our students, what is three-tenths times two-ones? The answer, six-tenths. It's

the only thing that fits in there perfect. And then over here, I go four-ones times four-tenths, it's going to be sixteen, right? Tenths. Here is where the language is forced again, three-tenths times four-tenths, in your head you're thinking twelve-hundredths, well, good, because there'll be twelve little hundredths down here that fit in perfectly. I like to show it visually to you, because it ends up being a manipulative nightmare, okay? Sometimes, teachers believe and then this, teachers do it first. I'm okay with this set of manipulatives. If you can find a large, if you can find a larger size, bigger squares and blocks of hundredths, you're going to be better off. That way, the students can physically move them to see the difference. But if this is one, what you're trying to show is, this is a tenth, and this is a hundredth. By using size proportional materials, it forces the students to say it accurately, eight-ones, six-tenths, sixteen-tenths, twelve-hundredths. And then they've got to added up. And they have to say it differently because if I'm doing sixteen-tenths, what else might be called that? What your mom going to call that? One and six-tenths or 1.6. So, we've got to make sure that that's done there. Now, are we able to make this adjustment? This is a language discussion. Are we able to make this adjustment? Who are we concerned with, who want to make this adjustment with us?

AUDIENCE: Parents.

DR. BRADLEY WITZEL: Parents, which is why a lot of moms and dads are here. Because the truth of the matter is, they were indoctrinated into a system of 0.1, 0.2, 0.3, one over two, one over three, not one-half, one-third. We are going to be forcing language on them, language that they're not ready for. And listen, I know a lot of people say we're not ready for it, yeah, but we're ready to get jobs again. So, we've got to start working towards better mathematics and better language and precise language where that is a step and correct direction. You know who the most available parents are who want to learn this? Not kidding you, pre-school. The pre-school and kindergarten parents are -- because the standard is new. Listen, we know this is way at home, I want you -- when you talk about twelve, make sure the child also understands that's one-ten, two-ones. Because by the time we get to decimals, we all have the general sense of decimals because we've made that error so often, say it with fractions as well. So, if I'm solving this, here's -- oh, by the way, that's the concrete side, CRA, here's the concrete intervention side to it. Here's the picto way of doing this. So, here, it's all size proportional. Once they nail the size proportion and they understand how to break it down, now we can start going through it and saying, "Okay. Four and three-tenths, and it's two and four-tenths." Remember, once we get here, they already understand the size proportions, this is a notation way of helping them through. So, when they're on a test and they're trying to draw it out, you know, so, they're going to start saying, "Okay. Let's take it down by the decimal. And that's made them, two times four is, right? Eight. Three-tenths times two is, six-tenths, they want to go four times four-tenths is sixteen-tenths. And then three times four is twelve-hundredths." Now, w can go back through it had a discussion where it fits. All right. And this is the abstract piece to it. Okay. So before we move to the next step, what makes sense out of this where someone maybe your potential frustrations with this? What some potential success we might have with this approach?

AUDIENCE: It takes a very simple multiplication and less things to solve three elaborate problems depending on the grade level.

DR. BRADLEY WITZEL: Good -- it's -- right? So, another words, work, we keep thinking -- okay. I'm going to re-explain, I apologize. It takes -- it takes a complex problem and breaks it down into a more simple multiplication. And how this ends up working for us is because our students think -- honestly, if a child says one 1,234 times 3,568, he goes, "Oh, my gosh, this is going to hard," rather than, "You know, I've got a couple of problems to solve here. If I grid that out, it's just as complex but I now have a graphic organizer in order to…" Right? That's the first thing you should do. Even if it gets out of order, it works. Now, I'll make a caution here, if you think this means a lattice, it's quite the opposite. I'm not anti-lattice, but lattice does not teach multiplication using place value. And the standard says that your algorithm must be based on place value. So if you're using lattice, you're undercutting place value. So, in other words, lattice is a good intervention for a student who has mastered place value but doesn't see where his numbers align. It does not work for a child you're still establishing place value, is that fair? If you ask me to show lattice, I'm going to tell you no, because it's not the goal. That's more -- I'm not kidding you, I'm pushing more Tier 3, a very unique, small problem that not many students will have. Okay. What sort of other positive of these kind of approaches here, this CAR approach? Not just with the place value, but maybe even with the popsicle sticks and all that. Yes?

AUDIENCE: [inaudible] understanding of what's happening instead of just following a process, taking some [inaudible] and understanding something…

DR. BRADLEY WITZEL: Now, the -- and I think the visual helps them see a property that we're so used to putting parentheses, and little faces and pumpkin and lines here and under, and now he's trying to go, "Oh, how does it fit here?" "I don't know, just first outer in or last? Which is the outer? Is it outer here or outer here -- now we got a process. Now we got a property, it's laid out. Give me some other ones, some pros now, let's -- okay. Let's do some cons with some confusion we're going to have here.

AUDIENCE: From a high school perspective as it gets come in opposite of high school and that for some teachers are not often introduced on this type of steps and so we're still accepting them, to give it a way you've always wanted.

DR. BRADLEY WITZEL: The traditional -- where is my traditional algorithm? And traditional algorithms are good, but we just kind of teach the properties behind them. So, now, as a high school teacher myself, I got to tell you, okay. Now -- and I like our model of our country for some of it, right? We need certain levels of mathematics in order to teach that group. The problem is how many levels of calculus will it take in order to teach Algebra 1? I'm not kidding you, sometimes we get so far beyond, we lose sight of the students' level. So, I actually called for if you are a high level person, start studying elementary mathematics. And I'll put it the other way, if you're a good elementary person, start studying high school and college level mathematics. You want to see where your stuff is going, high school people, you want

to see where your stuff is coming from. And it's all throughout because the goal is not to be best at your subject, I'll give you mine, I'll make fun of me and then I'll make sure that I let you go. Or make -- or make fun of me or let's say it's someone like me. So, there's a young teacher, an algebra teacher, he gets a lot of success early, he gets a lot of prayers. He's up for teacher of the year. Big district, lots of prayers. So his boss is very bright and nationally renowned, his boss is very smart, "So you know what, I want you to go visit the middle schools with me." So the teacher goes along and says, "Great. I can't wait to tell these middle schools how it should be run." Okay? And the smart boss dumb young kid teacher. Okay. So they got out to the schools and they get to the school which is, "Oh, by the way I, we're just going to sit in the back and watch about half a day." "What?" "Just watch." So they watch lesson after lesson after lesson and the boss turns back, the department chair turns back too this young teacher who's really good and really talented at motivation and says, "What did you notice?" And the teacher went from feeling good to shoulders are a little more hunch down. Says, "They're really good." "Yes. They are the exact same population of students you have. What do you think? Why do you think your success is where it is?" And the teacher turns back to the boss and says, "I've been thinking it was my awesome instruction when the truth of the matter was the students came to me prepared. If they had not had that background, none of my instruction would have made sense." In other words, if those progressions make sense to you and only you, they won't work. You got to convince and I don't mean convince as in please I mean convince as in, look at the standard, let's interpret this together. Here it is at the Fourth Grade level, let's follow through to the Fifth Grade level, the Sixth Grade, the Seventh Grade, the Eighth Grade. There are no tricks allowed. It is exactly as it's written. But that's going to be so hard for the children. No, it's not. It's going to be hard for us because we learn through a series of tricks and if we had good working memory, guess what they told us, "You're really good at math." Why? Because I can repeat exactly what the teacher said on a piece of paper five minutes later so I must be brilliant at math and the truth is that was not being brilliant at math, that was being good in working memory. What we're trying to do is approach it differently so we will not only saying to Gifted Ed or good working memory. We're trying to give the Gifted Ed groups to the students who had those unique abilities because right now as a dad with two kids in labeled Gifted Ed, I have know idea if they're gifted and children, they know this, I love them and they know this, "Are you gifted Laurel, Caroline?" And their answer is every time, "I've had a lot of preparation." And that's not always coached, it's because they see a kid right next to them who's just as smart if not smarter and they can do it -- and they get -- but the reason why they -- my kid can do it, he's been prep. She's been -- she's learning this. Gifted in our country is those who have good working memory and good attentions skills. We've got to get beyond it. How many of us struggled in school, succeeded in life? All right. What does it say about ADHD? They make horrible students and impressive adults. We've got to go through and find out what meets our -- what meets their needs better. All right. With grade [inaudible] are. CRA delivery, the steps for CRA we actually break it down to CRA math. My buddy Paul Riccomini calls it CRA-math. Whoa. I don't like that. I think of crawfish and my Louisiana buddies but just CRA math, choose a math topic break down the procedures and get rid of the tricks.

Well, I know it's whatever it is that you think is hard for your students to learn whether that is long division which has been correlated -- long divisions and fractions have been correlated to algebra success, the highest correlation in algebra success comes from, yes, attention memory and all that but it really is built from those bottom line basic fractions division. So we've got to set all that up and we've got to go through and make sure that they understand all the building up to work, learn the real procedures and it was a trick then you go, "I don't know why this happens. Find another way to teach it." Next one down here, match the abstract strips -- steps, easy for you to say. Match the abstract steps to a manipulative. Now, I'm okay with expensive manipulatives but you can tell that once that you're probably going to member here the most are things like popsicles sticks, things like cardboard paper that has positives on one side and minus on the other. I'm okay with integer chips, the problem is with integer chips, there's still a sign that's not being address in there, it's positive three, it's negative three rather than it's just three things that are negative. Okay? So think about the easy simple things that we can do. Then start teaching -- make sure your lessons are arranged. If I'm five steps here, I should be five steps here and five steps here. It is CRA if the steps match. It is not CRA, we have negative effect size if you decide to do C then R then A. In other words, you teach concrete one way, pictorial representation and other and then abstract the third different way. Kids with memory problems will not do well at that. You've asked to memorize three different ways to do same problem. And I know a lot of us say they should do multiple ways to approach it. I absolutely agree but they better master one before they move to number two. So, that's where CRA needs to work. C-R-A is not effective but CRA is highly effective. We've got a lot of programs out there that's saying they're CRA research based and they're not and that's just the simple truth. Teach each concrete representation abstract, teach each sequence step to master it and then the last one, teach some generalizations, get them to talk the whole way through. Now I'm going to give you -- why do you think CRA might be helpful? Oh, boy, we got time issues. Why do you think CRA might be helpful? For whom would CRA be helpful?

AUDIENCE MEMBER: Teacher.

DR. BRADLEY WITZEL: Teacher. Why the teacher?

AUDIENCE MEMBER: Because then we have some way to [inaudible] transform through the students.

DR. BRADLEY WITZEL: I love -- actually, when we do CRA lessons you might find that I infuse a lot of content. Well, it looks like some instructional strategy, it forces us through content. So it forces the teacher through content. They can't skip steps because there's nothing magical that can happen. May -- things can't disappear. Like in division, things can't suddenly like gets -- get smaller. I have kids who think nine divided by three, it gets smaller and the truth nine divided three is what? Three groups of three. All you're doing is breaking the part nine into three groups all the time. So it forces us through the steps. Who else might this be helpful for and why?

AUDIENCE: [overlapping conversation]

DR. BRADLEY WITZEL: Not all at once. Slow, slow, slow, I can't hear you all. How about kids with memory problems? They have motor memory finally to reach back on. We have found that kids during tests do better with this. Our first case study came from a young lady who graduated without her Algebra 2 but she needed that third math to graduate so she -- teacher thought that he gave her D in geometry and she was done. The problem is she needed Algebra 2 to graduate. She's down in Florida so she came back to us in summer school and said, "Help. I was at University of Florida. Help." So the councilor came to me and said, "Could you be involved in your pilot? "I said no it's -- that's strategically done but I'll just work with her." Oh, my gosh, Courtney was amazing. She showed up everyday at the library, we work for an hour and half. We got through Algebra 2 with the -- with somewhere around like a C, C+. Please understand that that's the highest grade in math she's gotten in years but what she said is -- the whole time during the test, her test were hieroglyphics. I mean there were circles and lines and dots and squiggles because the teacher asked, "Well, what are you doing here?" She wasn't going to say it three groups of X. You know what she'd say? Well, I have three cups of X. She was at the concrete level of understanding. She can write it as if she was abstract but she stayed concrete. She was not going to get beyond and she was still successful in her solutions. We've got her Algebra 2, we've got her to our community college. Oh, by the way, I just want to make this point too. This is one of our Gifted Ed students. I don't know why he showed up to our intervention. We are happy to have him. You know this is South Carolina right to the toes, you know that's South Carolina. So, I mean we wear flip flops to our teaching but he came to our some interventions and she was trying to teach the angle theorems and he's said, "I'll just do it on the board," because he's gifted, he'll just catch it. But you can tell where the lesson had end up going. My daughter was outside with sidewalk chalk on the -- on the steps in front of the school because it's summer. I mean, I got kids and so she's outside the protractor sidewalk chalk and we sent Matthew and the teacher out there to work it out. As he's working through he goes, "Oh, my gosh. Now I see it," because he was measuring angles. "I get it." Don't hesitate. I thought a class once of 52 students in high school. Don't hesitate to bring them to a larger venue and get them on the floor graphing it out. My favorite lesson of all time is bowling toilet paper for simulations equations. We had red -- it's around Christmas, we had red toilet paper and green toilet paper. And we would bowl through the lines and with every one, they're intersecting because your floor in school is all tiled, right? So the grid -- the quarter plain is already set for us. Be unique with it. So I'm going to give you an Albert Einstein on this. Says, "Whatever your difficulties in mathematics I can assure you mine are far greater." You on the line sense background, right? He said he struggled in literacy but he also struggled in math. What he had is a conceptualization so when the few people in our world, he said, "I loved algebra. I never could memorize all the arithmetic. Algebra explained everything I couldn't memorize in the first place. Love it." All right. Guy, we got about 10 minutes. What are your questions? Do you want to see extra demo once on things? Yes, Bill.

BILL: I'd like to know what you think about or the -- or the -- or the -- what are the programs that you would recommend?

DR. BRADLEY WITZEL: Wise. That was very wise because you knew I couldn't address a program. If you noticed Bill, Bill said other programs I can recommend.

BILL: Like for example.

DR. BRADLEY WITZEL: Okay. Oh, boy.

BILL: In New York.

DR. BRADLEY WITZEL: Okay. Oh, yes. Oh, well.

BILL: [inaudible] and the state of Georgia has frameworks that are available and Utah has an integrated high school program and it just came out with Seventh and Eighth Grade.

DR. BRADLEY WITZEL: I know metrically, the three -- okay. They have mentioned three programs there's one out of state department of Georgia, one out of state department of New York and that's to engage New York, it's getting a lot traction, people are excited about it and the other one you gave to Utah, right? And Utah and I've worked there over the summer as well. These are good programs but they have no research support directly. So, I -- they are good to supplementary program. They're not all CRA based. They're very engagement based and they're very practical applications but there's no research to show that these activities specifically have helped students learn. So, at this point it can complement your curriculum. I caution -- because I work with districts who are doing these things. I caution them on making up their curriculum. Let someone else be the guinea pig. Do you want to be the guinea pig, stick to the program that you have and if you think there's -- it's too passive, that's for somebody's activities are really good at it. As far as CRA goes though, none of them really build immediately to CRA. You would have to rearrange the lesson to meet CRA. There's a caution for CRA. US Department of Ed and Institute of Education Science said CRA is a great intervention. You may look the distance, oh, my gosh, I'm doing that for instruction and I got you. I know where you're coming from but the research really is stick to -- stick to the higher level rigor of math before you start breaking it down the popsicles sticks. Popsicles sticks are a way that you're going to break it down for the students' struggles. They still need that core instruction and conceptual understanding of size quantity upfront first. Questions, yes?

AUDIENCE MEMBER: How -- our district is in a lot with Singapore math and…

DR. BRADLEY WITZEL: Uh-hmm.

AUDIENCE MEMBER: …I see a lot similarities between the two are being based off [inaudible] ideas?

DR. BRADLEY WITZEL: Again, Singapore has -- Singapore's math program has been interpreted by a -- there are -- at one point, only one company, I was asked to be a spokesperson for one but I think what the three different companies who are trying to rearrange Singapore. Do -- are you following me where I'm coming from?

AUDIENCE MEMBER: [inaudible]

DR. BRADLEY WITZEL: Okay. There was several companies trying to say that they're Singapore, there's only one country Singapore and they only have one curriculum and it's not English Language. So what they're trying to do is they're trying to rearrange it to match the US desire because we -- why don't textbooks have the -- why do textbooks have the tricks and not the actual math that goes with them? It's because during textbook adoption, people are afraid that you'd open it up and go, "Oh, I don't get this." And they'll be dump for textbook adoption. It's -- there's pros and cons to the capitalist approach. Singapore has one exact program and that's all they use. So, I give you caution that the -- each different interpretation of program have not had the length of research. We don't know the outcomes. Also if you say I'm going to Singapore and the next grade teacher does not, you're going length based diagrams and no diagrams. Length base diagrams have research. Stick to the length based diagrams. So in another words, I'm not endorsing Singapore but I am endorsing the use of length based diagrams. So, in other words, there might be a program out there that's not called Singapore content but they use lots of length based diagrams and it is research supportive. And I -- yes. Institute of Education Science which is we have Department of Ed. All right. There's no IES Bowl, I can get an Armed Forces Bowl. There's no IES Bowl but you can go to ies.ed.gov, they're not allowed to advertise but you can get all the research guides there, all the date there, they analyze US program outcomes for specific interventions and overall -- overall curriculum and you can see the outcomes there. Those length based diagrams are incredibly important. But now -- but again, but now I hope you see that there's a concrete that can preceded but we really want to get to the abstract. Okay. Other questions? Yes. Kimberly?

KIMBERLY: What would you recommend for parents…

DR. BrADLEY WITZEL: Oh, I love it.

KIMBERLY: …who are still in the old days…

DR. BRADLEY WITZEL: Quite good.

KIMBERLY: …that, you know, like -- I mean, I don't necessarily would have to go back pre-school to start learning all this. What do you recommend for those [inaudible] all of the sudden Fifth or Sixth Grade level with their kids are saying, "Huh. No way. It's…"

DR. BRADLEY WITZEL: I'm so happy you ask that. Now Kimberly's question was what do we do for parents, why --what's -- what should we ask parents to do, parents who are informed and have that desire to step in? Well first is we have a curriculum gap going on. We have a standards gap. The -- why did I do all this early with my child is because with common core conquer is be developed and they were having the discussions with National Math Panel and the NCCSS, right? The Chief Councils of states cooperators when we're having our conversations, I went -- by the time this roles out, my child would be in middle school, my oldest child, which means there will be a curriculum gap kid. They're going to have the long background to leaving Sixth Grade and guess what ends up happening. So I thought them based on this. So in other words the kid went to school and knowing it one way and then came home and dad who is always wrong. Dad kept trying to teach me these other ways, I'm so confused. Now she gets

the connections. So here comes -- now you got a curriculum gap kid. You said, "I don't want to go back," but we're going to have to go back and infuse that language. Number one, infuse the correct math language. We've got to change our language and we're going to capture our souls doing it. It's going to say 1.2 miles to the exit and you're like 1.--oh. One and two-tenths miles and you're going to force the language on them and that your kids will be like, "What are you doing mom?" Just say it with me, one and two kids -- you're going to have to force the language. The next one is there are videos out there that you could watch. Some state departments are putting out videos so you can watch the instruction. There's good -- there's good general approaches like Khan Academy have some good stuff out there but when it comes down to the accurate language, the use of the ways and common core, look to a state department approach because state departments have kind of a general approach proof what we want everybody to say. I know that Pennsylvania has talked about it. I know that Massachusetts has done it but they -- they put a password protect on it, but there's some other state departments that do that. But again it's going back through and watching these videos making sure you repeat that stuff over and over again. Afterward, I can talk about programs that could be use, I'm just afraid to say the program now because it sounds like I'm endorsing that versus another when in fact I'm not. I'm just telling you there's some have these really good videos we could watch. Sir? Okay. We got time. We have time for any more questions. See, when I say that, that was -- that was kill. You're like we're going to run the lunch early. All right. Yeah. You, what's your question?

AUDIENCE MEMBER: So, you have a child who's in Tenth Grade and they're now -- and her district constantly keeps changing.

DR. BRADLEY WITZEL: Oh, okay.

AUDIENCE MEMBER: So everything now then switched over to integrated math and only few kids that have honor or [inaudible] or in an Algebra 1, Algebra 2 so…

DR. BRADLEY WITZEL: And early because of their memory because they were able to get that information and have enough time to get some of that quick memory to learn all the background that they missed.

AUDIENCE MEMBER: So what…

DR. BRADLEY WITZEL: So what do we do? Okay. To the question.

AUDIENCE MEMBER: [inaudible] and these are [inaudible] cool and they get to college and they're all taking…

DR. BRADLEY WITZEL: Intro.

AUDIENCE MEMBER: …basic math.

DR. BRADLEY WITZEL: They're not getting credit and that's the adamant study of 2006 that you're -- if they're not taking any college course that is a college match course that is for credit to begin with, they will struggle to graduate with community college degree even a two-year tech degree. And I'm talking about -- I'm talking about diesel tech, I'm talking about plumping prep, I'm talking about all the things that

we need this country to have and first I want to make sure I step in my sole box, I'm coming right back. Biasing college bound is the only way to go is a mistake and the truth is we got to make sure we don't screw bound, post secondary should be the push so they get the technical degree. So they do the things around this country that help this country work so strong and be so successful. [inaudible] to explain that, right? You fell so many times before you succeeded. That's the kind of thing. Okay. Now back we have a district that has been in searching and they have chose this curriculum and that didn't work and they chose this curriculum or a big district you might have issues with the text book reps are upset at you and you didn't look at our text so they made you by this other text to [inaudible] these gaps. Well, here's the good news, the first part is that the textbook authors who did at Grade Three are not necessarily the same textbook authors who are Grade Four. It's usually a team of people that write all of them per publisher. So already you're going to have a mismatch even if it says the same name on the front of the textbook, you might have some mismatches throughout. So, what we -- okay. Some know didn't have time in here but some of my friends know that I like to do this, take some sticky notes with teachers in your school district and label now K-1, 2, 3, 4, 5, 6, 7, 8, Algebra 1, Geometry, Algebra 2 and then have them list out what are the areas that your kids struggled with and walking into your grade level. So the kindergarten groups can say and for mom and dad they're not coming in with this. So we end up having to spend time on it even though it's not even my standards. So what you're going to do is you're going to lay it out first. This is just a screener. This is a district wide teacher based screener. By the way teachers are great at general screening. What you see and perceive is typically the right answer. So please know that. Most data points to ask the teacher, she'll tell you where the problems are coming. All right. But once you get this you're going to see some things repeated. Now you mentioned some programs to me and I'm not repeating those programs but multiplications is going to stand out and because multiplication is weak, stand out because long division weak, fractions will struggle and then algebra will struggle and so we've got a string of problems are set up here from the very beginning. They didn't memorize this, they weren't told to and here comes the chips and the dominos keep stacking down. So now we have to go through with older students and say what are they things we can clean up, what are the things we cannot so in those regards, focus interventions. If you see something that's repeated the whole way through, start trying to clean up those interventions, great. I have seen this now for four years, that becomes an intervention for Fifth Grade. That's the problem that becomes intervention now for Sixth Grade. There's still Algebra 1 but they might need interventions on the side for fractions and integers and I would make that integer for -- if they're weak at multiplication, don't just do multiplication get with them. Do multiplications with integers mixed in. That way you're trying to build up to the next level. So again it's a quick way to see the answer but you have longer discussion of ways to gear an RTI 9 Program to make sure these interventions aligned with everything. But you got your hands -- in other words, there's no quick tricks today. You really have kind of a five-year plan you're going to have to lay out. Other questions, comments? We're out of time. Isn't that sad? All right. If you are crazy and love math so much, you can hang out a bit. I will show you solving simple equations. The hand outs are up

there, the examples are there the US Department of Ed shows videos for it. I just want to make sure that if you need more examples, I provide you those examples as well. Well, thank you for you and applaud for you. You applaud if you hated me as well. Thank you.