

JARED CAMPBELL: And good afternoon. My name is Jared Campbell. I'm an Educational Consultant with PaTTAN. And I have the pleasure to introduce for the second time today, Dr. Witzel. How many people were in the morning session? You see, that tells you how good this afternoon's going to be because we have had many repeaters. Bradley Witzel is an award winning teacher and professor who works as an Associate Professor, Assistant Department Chair and Education Program Coordinator at Winthrop University. Dr. Witzel has written several dozen research and practitioner articles as well as seven books including the best selling "RTI in Math", and recently published "Building Number Sense" through Corwin Press and "Rigor for Students with Special Needs" through Taylor and Francis. He has developed and presented a dozen educational videos as well as delivered over 300 workshops and conference presentations. He was an elected member of the Smarter Balanced Assessment Consortium, Accessibility and Accommodations Work Group, and is a governing Board Member of the Southeast Regional Educational Laboratory. A research hub funded by the Institute of Educational Sciences. Dr. Witzel currently serves as the editor of Focus on Inclusive Education through the Association of Childhood Education International and recently served as an author and panelist on the IES Practice Guide Assisting Students Struggling with Mathematics. And as an invited Reviewer of the Final Report from the National Mathematics Advisory Panel. It is my personal pleasure to introduce you -- to you today, Dr. Witzel.

DR. BRADLEY WITZEL: You have to understand I've [inaudible] I don't have any clue what he's talking about. Those are just a bit fancy multi syllabic words that makes us think better of each other but the truth is, I think my most important title is father. And so I am -- I was a high school math teacher and a Special Ed teacher. I was actually a Physics Major, and that long story and kind of discombobulated. I was convinced by my physics friends that I should probably become a teacher. And then when I teach Physics, I wanted to make change. And I thought by teaching physics, I could have people excited about the math and sciences. So they actually encourage me to become a Special Ed teacher. They said at James Madison, I was the first person they'd ever seen from physics to Special Ed, they had no idea exactly what I was talking about but I -- it made sense to me. So with that dual background I had the absolute joy and privilege of teaching the struggling learners' high level mathematics, and I got to tell you, it was a great challenge. So I'm explaining this background to you ago why we call that number sense then. Now, a lot of the times if you heard the earlier sessions, they like -- like some of the familiar faces here. And I hope you've intended to come back, it wasn't like, "Oh, my gosh, he's awful. It's got to be better the second half." I mean, I'm with you. But the reason why it's so important to understand that -- whereas the question, did they mess it up? Did PaTTAN mess this up? Did Pennsylvania mess this up? Because they have -- they have ranted on the high school stuff first and they started early childhood second, I don't get it. And the point was, high school teachers will come to a high school event but they won't know that they need to go to a numeracy event until it's laid out for them. And I'll flip it. Early childhood, people go to an early childhood event, but if they put the word Algebra in front, they go what? Scared. So what we really need to do is try to realize that algebra is nothing -- for the warning. Algebra's

not mystical, it's not magical. Probably more complex than a regular arithmetic. What we're going to do now is we're going to help the foundation of skills and build into these. So why is this near and dear to my heart? Why do I now have books on this stuff? A lot of them who's doing research on it got babies. So if you got babies, you understand passion. My babies are getting a little bit bigger. I have a middle school baby and that is my true baby, and then I had a third grader who think she's a Middle School. So there's a difference here. And they had convinced me to get some processes taught early. I was involved in earlier discussions on the Common Core when they were developing through with the Focal Points. From the Focal Points, I was involved in National Math Panel. So I got to see a lot of the early stuff before it happened. So I get to guinea pig, my kids. Now, I got to guinea pig, other people's children too and I will not show their videos. What's on the recording just doesn't work. Those copyright verbiage issues and so, what I'm going to do is I'll show you videos of my children going through this, and you're going to hear the language. And they hate some of it. Those of you who've seen the videos of my little people, they hate some of it because now they're older and then looking back like, "Oh, you're making fun of me." Well, kind of. Yes, yes, kind of. Well -- but we're going to explain why it's important to see where that comes through. So, again, you got a little bit of my background of what my interest are as a high school teacher stepping into your world. What I cannot duplicate is the passion that you have with little people. How you could herd crickets better than any -- anything I understand. Okay. I mean, they're chirping, they're jumping everywhere and somehow you draw back to -- in an area that I just don't have power to do. High school kids, easy. Snap. Fall in line. Do this, do that. And what they do, just fall in line and walk, right? That's different with little people. So let's start. Where you are, moms and dads, let me see your hands. And you could be dual, mom and dad, and teacher -- moms and dads. Anybody have young children at home? Okay. If you have young children at home, I'm hoping that some of this, and some faces out there that I can talk to. I hope some of these hits -- hits where the changes could happen even at home. Because the Common Core did not come along and talk about we need to change schools. Common Core is a cultured shift. It is an approach to math shift. It's not, you know, if fourth grade teachers did this, magic things would happen. No, it's -- we need to have a US societal change of how we approach this, okay? And [inaudible]. There will be -- will be a day that we pull out an abacus and it makes sense to a child, and hopefully to an adult. But right now we got to make that shift. All right, other ones here, do I have preschool -- do I have preschool teachers? Preschool Special Ed? Going once, going twice? Okay. Kind of. I don't get the kind of but.

AUDIENCE MEMBER: Kindergarten [inaudible]

DR. BRADLEY WITZEL: Good. So let's talk about this. So early childhood, let's go kindergarten through third. Kindergarten, oh, yes, Kindergarten through second then. Okay. Where's my third and fourth?

AUDIENCE MEMBER: Third and fourth.

DR. BRADLEY WITZEL: Fifth, sixth? All right. Worse, there's too many middle school interlopers. With my middle school teachers [inaudible] find your way here, huh. I got you. Do I have high school

teachers? Oh, my gosh, high school people. I love you. I'm glad you showed up for this. Who works with students with high incidence disabilities? In other words, typically higher achieving students. Okay. How about my low achieving students, who works with low achieving students? Okay, there is a distinct difference. The research would be the same group, we're talking about kids and at-risk or with disabilities. They're very different on how you approach one versus the other. So I will try to bridge that gap but good luck to me because, I mean, think about autism alone. Someone says, "I have a child who had -- who's on the spectrum." And I love on the spectrum. I'm on the spectrum, you could see me through visible light but that's a different world. But they say they are on the spectrum and I -- again, get my head around and try to figure all that out what that means. But when they say that I don't know what that means because I have lots of kids, I've been told one of my daughters could be on the spectrum and I said, "Yes, so could her dad." I mean, it depends on the day that you catch them. So in that regard I got some students who are on the spectrum who are high achieving. I also have some students who are identified with intellectual disabilities who later on are no longer identified with intellectual disabilities. Now, I got to tell you, the IQ test I thought was supposed to be stagnant, static in fact but things move. So we've got -- we're going to have discussions on where those movements occur. All right. Do I have -- where's my school psyches? Oh, good, my assessors in the audience. We're going to be using you, and this -- where's my admins? Which could be math coaches or administrators as well. Okay. Different districts. I know I'm missing people. PaTTAN. Where's my PaTTAN people? That's right, I love my PaTTAN people. They're over -- it's been a decade now. Can you believe, it's been a decade? I don't know why you people invited me. Okay, here we go. So here's your warm up for the day. I only have two symbols up here and I want you to try to interpret these two symbols that throughout which of these answers align with those symbols. I give you three examples, and you've got to figure out the rest. Fair enough? All right. Now, by the way some things could be repeated. There might be two versions of four type of things, okay? So I'm going to give you about a minute on it. Try to figure out that. At least get A, though. Any magic? Anybody figure out some of the codes? It's not a full minute yet. For example, let's start here. And I love this about textbooks. Textbooks would come up with these things and ends up the modeling one only helps the students get through questions one to four, right? It's the ones that they didn't model for them that actually takes you to the rest of the page. So let's start with B. What's B?

AUDIENCE: One.

DR. BRADLEY WITZEL: One. Okay, what's D?

AUDIENCE: One.

DR. BRADLEY WITZEL: One.

AUDIENCE: One and a half.

DR. BRADLEY WITZEL: One and a half. I love it. B and D are both one. Are they both [inaudible] As that we represent one? In our -- in our language system, in our writing system? Yes. We do it verbally,

we do it as a dash, we do it as a little hooked one, and so and so forth. Oh, we blew off [inaudible], right? So the B and D are both one. Let's see, one, three, five, seven, nine. Give me on this one. one, three, five, seven plus one is 8. One, three plus one is four. How many people have just caught or hold on. How many people just got -- now you got the whole system down? You see, examples help some but not everybody. So what else is needed? Here comes the more explicit. When I see the number symbols, I count by their odd numbers beginning with one. When I see this [inaudible] wannabe rhombus, I add one to it, whatever the number is. So this would be add one, add one which means the number is...

AUDIENCE: Two.

DR. BRADLEY WITZEL: Which means down here it's one, three, five, seven, nine.

AUDIENCE: Ten.

DR. BRADLEY WITZEL: Ten. See? So you there are different theories of modeling that different students need as well. Adults are the same way, right? How many of you think -- went to school with -- and Math was taught to you like drivers Ed was taught to you? Get behind the wheel and I'll yell at you during the time, right? Don't worry, I got a pedal over here. I got a brake pedal because you're going to mess up a lot, right? That's kind of how Math has been taught, and we got to make that change. So in this case, by the way, is this hard to do? Not if someone teaches you. But I'll make the shift here. One more time about digits, there are 10 digits put in multiple formats that could mean millions of different things. And that's pretty complex but it's not as complex as literacy where we have 26, right, different letters that make -- depending on your dialect, 40 to 46 to 52 different segments, so in that case, language and Mathematics, a lot of people like to link -- lock them together, they're actually quite different. And the research behind this has become quite different where a lot of people say, "Well, I've worked in literacy, let's do it in Math." And then we'll say, what works in Math, let's do it in Literacy. No, we don't hear that in this culture. All right. Here we go. Well, the sky is falling begin -- let's talk about the sky is falling. We found that students in grade four -- well, this is -- and by the 2013 report just came out very similar. We have about 15% of our population in grade four below basic. By the time it's 8th grade, though, we have 25% and then went over a third of our students below basic by 12th grade. So let's describe the four levels here. There's advanced which means you're doing very well. There's proficient which means you understood the standards and are ready -- plainly ready to move on, then there's basic. Basic means you got some gaps, you kind of get the standards but you've got something you failed on but we're ready to move on for the most of it. Then there's below basic. Below basic means you're not ready to move on. You're almost at a standstill in your learning, you needed -- you needs -- if basic means you need intervention, below basic means you need serious intervention time put into it. Okay. So when I describe a group that is below basic, that means this is a scary group. This group, oh, my gosh, we got major changes that need to be done here. So what scares you about these numbers then? If these seems are the consensus that means below basic, what scares you about these numbers? They

go up as the students progress, why? Because the work gets harder. So tell me about Math, it's difficult from almost every other subject.

AUDIENCE: It builds on itself.

DR. BRADLEY WITZEL: It builds on itself which means if I miss something in first grade and I'm going to state it simply than I did this morning but the same push. I can actually skip -- okay. I've got principals go, "We got to get our scores up. What should we focus on to get our scores up in first grade?" And I say, [makes noise]. And she goes, "Well, don't you know?" I go, "Absolutely. I can tell you exactly what's on that test. I can have you ace that first grade test. I can probably prep you on the six things that dominate that test to get your scores up." She goes, "Really? But I won't do it." "But wait a second, why won't you do it?" Because it will look good in first grade, your first grade scores will go up. But the problem is you're going to be sacrificing second, third and fourth. And their scores will either go down or stay the same. The goal is not to have the best first grades here, the goal is not to have the best third grade student. The goal is this long term progression. It's why we're going to hear about number sense. That's why we don't call it, all right, early numeracy. We call it number sense. All right. Tell me one thing that scares you about these numbers in general?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: They're high. And much higher than other countries. We have a -- it's -- our scores internationally have gone up. That's -- we need to have some praise here. We've gone up. I think we're now like, instead of 23 in the world of industrializations, I think we're up to 22. Guys, it's a growth, right? Let's take it. Growth patterns. We love it. No, that's not good. We still have -- by the way, we have almost by little group development. We have a group that is high achieving, they're very small but they're among the top in the world in Mathematics. And then we have another group, the general population. Our average general population is lower than those other industrialize nation's general populations. That's what we need to have our discussion on. All right, real world performance, National Math Panel. This might not shock you. Most adults who have to know and use interest on loans, most adults do not understand it. At about accruing interest or when to take points and when not to take points, right? You sat in front of the -- of an -- of somebody, a mortgage council, who gave you an interest loan and said, "Well, here is the options for you." Guys, you understand they take commission, right? You get this, right? You should be on the other end with your piece of paper, and your calculator, checking what they're doing. Whoa, whoa, whoa, that's not the percent. I thought you said that there were no points, you just added a quarter point to this, and when is that important and when is that not important. And I'm not saying the world is dishonest, but if you don't know your Mathematics, as Moses said -- this is Bob Moses back in 1985. Different Moses back in 1985. He said that, "If you do not understand your Mathematics, you are missing out on Civil Rights." He says, "Teaching Algebra is a Civil Rights concern because those who miss it will not be able to achieve later in life. There'll be -- they'll be entrusting to others what they themselves should do." Other ones over here that might have caught you,

maybe you don't want to calculate miles per gallon, right? Like, those who cannot maybe they should not. Most who can do tips, 27% of 8th graders could not correctly shade in the third of a rectangle. They did is, they gave a rectangle and they have it parsed in the -- they had it split in the middle, it said, shade in the third. So people are like, "Huh?" Or they have like a part here and make a little slipper here, they'll just shade one of them in, nothing that they need to be proportional and equal sized. You get the point? They just -- they're following the procedure out without understanding what they have been asked to do. And last one here, welcome back to adults, 45% of adults could not divide fractions. All right. I'll just invert and multiply but -- and again, that was the last session. We're doing a little different. So here we go. Where are we going to start? Where are you making the change? Most of us think adults -- and I'm going to tell you, they have a better chance to making change than we do. In fact, as I show you something [inaudible] please understand, you're going to go through this, go, [makes noise] "I don't know if I like this." You got it. This was not designed for our enjoyment, this is designed for their performance. So we're going to have specific things that little people will do that will gear and help them perform later on. Things that you and I will be like, "But that's not the way I learned it." And the answer to that is, "Good. We're not doing well." Maybe you haven't noticed but we're real low in math. We used to be the top of the -- of the world. The problem is they said the rest of the world, right, like were dropped? We haven't dropped too much. You know what happened? We just stayed. So our sliding growth doesn't match their growth. They found ways to meet the masses, we already started by getting the masses, that's what we started so far ahead but your little turtle will get caught up by the rabbit. But now what are we going to do about it? Thanks. Because I don't know who they are, but they look really cute. Okay, here we go. The research, the check -- please check my work on this at all times. Don't ever go to a talk and say, "Well, according to Brad." "Well, that Dr. Witzel said it." Please understand that, sure, it sounds neat and I'll take all the credit. It makes me feel good about myself. The honest truth though is, I'm reading the research, and the reason why I brought on these panels is, I think I'm okay at clarifying the research in describing when it make sense and when it doesn't. So these are the groups that you're pulling your data from. The IES practice guides. If you look up IES.ed.gov, it's the US Department of Ed, we're going to focus on one of their practice guides today, on early numeracy. Which means we have research effects that are so strong, the US Department of Ed is actually setting out guides now, do these things please. This will change the nation if you do this. Has everybody ever seen these? Why do you think there's only four or five hands here? And I make the joke but please know that US Department of Ed is not allowed to advertise. We do not have a budget for it. They keep yanking things down to their websites because of sequestration and money, but the truth is they're not allowed to advertise. We have the Armed Forces vault, but we don't have the Department of Ed vault, and that's what I'm looking for. We got to make things known. We got to start finding ways how to get to this research. Although, it's out there if you have not read the National Math Panel, please -- the final report, please do so. If you've ever seen the Center on Instruction, they're one of the largest like RAND Corporation, that's one of the largest education research groups. You can find their work. Two other non-profits NCTM has come around after

losing their standards. They have made it move, they say, "Well, maybe we should work with the Common Core a little better." And they're doing a good job. I mean, so if you've been turned on by them, their back. Okay, and skip federal and I'll go back. And the last one here, there are some good new assessments. And psyches will tell you, good new assessments coming out at Number Sense. And we're going to talk -- I'll share with -- one with you today. I think we even downloaded it. I think it was on the website. So here we go. IES practice guide. I'm going to share this then we'll get to little people. I want to make sure you understand what minimal level means before you think, "Oh, I shouldn't do that." A high level of evidence means we've had a lot of studies and they all say the same thing. Moderate level means, we've had a good number of studies, they all say the same thing. Minimal level means we've had a couple of studies, they all say the same thing. This is brand new research. I hate to tell you this, Math educators had not been good researchers. They have not. They don't run their stats well. Think about that, okay? Just stay -- stop there. Yup, you got it. So in this world, it's not as if minimal level means it's not good. It means that we don't have enough research to throw the big step at you, but we've got enough to tell you where to get started. So some of them -- and I hate when people read, so I'm going to do that, I'm a hypocrite. So I want to make sure I hit some of the highlights here. One, number and operations are important. Students need to know operations. I mean, [makes noise] yes, they need to know their facts. Okay. And these facts must be progressed from grade level to grade level. You can't approach a Math problem one way in first grade and say, "Okay, now we're done." And second grade, "Now, we do it this way." If you approach it one way, it needs to be tapped on and built from. You can't let the children do it one way then stop and switch on. They're not going to understand it. Anybody here had the privilege of teaching the same group of students three years in a row? Different grade levels. Okay. Okay. Not repeaters. Yeah, and you'll learn what you taught and what you didn't. I have to do high school, I apologize but I taught my group of Algebra, so in Geometry I said, "And now, last year we did this, so this year we're going to do this." And they're like, "We didn't do that." "Yeah, we did. Remember the polynomials. It's like the last..." "We didn't do that." It was a foreign language student. I had not scaffold him from one section to the next. It's very important that we set that gear up early so that builds. Next one in here. Patterned Geometry, data analysis. They're important as well. So in other words, you keep hearing for kids with special needs, "Oh, my gosh. I know what we need to do with a kid with learning disability, just more multiplication facts. That's all he needs, are multiplication facts." Now, no, he needs more. He needs application. He needs to know where this works. The other one that's in here, progress monitoring. How many of you here are already doing progress monitoring? Okay. You should be building a system in as early as possible of what their learning and how quickly they're learning it. And it doesn't mean that you're just doing [inaudible] fact retrieval. You got to have more details to that. All right. Children need to see the world is Mathematics. And we're not going to have evidence on this one. Okay, there's not a research study that says this. This is the obvious. If they only see Math during 15 minutes a day -- how many people. Okay. How many take French in high school? Okay. How many of you are still fluent? One. Why? But you took French ed, how many four years? How could

Frank be the only one? Why? Because you took it for 50 minutes a day, you never applied it afterward, and then you thought you never needed it for the rest of your life. If I dropped you in France right now, you couldn't find the bathroom, right? You're going to yell, "Baño, si?" No, wrong. My wife is a French-Canadian and my youngest daughter who still goes, "Mommy, keeps speaking Spanish to us, Dad." Exactly. I don't know how to answer that one, "I love you very much young lady." That's -- I thought it was German but that's okay. I don't know. The last ones in here is dedicate time each day to integrated math. In other words, don't have just a 50-minute math class when you're describing history, if you're going to be talking about the Winter Olympics coming up, it is a great time to reintroduce them to the Math behind it. No, not the number of man holes they got blown out or something like that, I'm talking about if you get into the scores and learn the tenths, will they have fractional scores, or you got places. And it's important to understand or pull data, that's a great data time because you can get first place, second place, third place. To a little person that means this, so some of you are upper level in elementary, it could be first place, second, third, fourth, fifth, sixth, right? It might be that gap. I might have a [inaudible] plot that explains it better than an ordered how many got it first or a histogram. We've got to start introducing them where it is everywhere in the world. USA Today does a good job, the newspaper of that front page showing that craft, now we need to talk about that level. So this is -- this is a practice guide. Look how little data we have, we got -- there's only one thing that has data and it's, you better teach numbers and operations as often as possible and do it the same way everywhere. So here we go. Trajectories. It's little, I don't want to write through all of it, I just want you to know the long term data on this. Number sense development tried from kindergarten to first grade, if I assessed them one way in kindergarten, I can actually predict that first-grade achievement. Are you surprised? When they enter -- guys, when they enter kindergarten, we can asses them there and I can predict how it's going to be at the end of the year. Are you surprised? A little? But it's not good, is it? Those who have been given a background by mom and dad before the age of five are going to come in better prepared. This should be no surprise because in nursery we have the exact same details. Which means when a child is born I guess he learns to babble. Some states [inaudible] giving them pamphlets, read to your child. Do this for -- a whole pamphlet of how to teach literacy to your child. There's no pamphlet that says what to do in math. That math is everyday, that you should be counting from zero with your child. That math, double digit numbers aren't spoken as thirteens and fourteens. That's an interpretation made. That's not the number. Right? There should be things that explain these to moms and dads. But we're not a numeral -- we're not a numeric society. We're not a STEM society; we just claim to be one.

AUDIENCE MEMBER: Okay.

DR. BRADLEY WITZEL: I'll continue. The majority of children in low or flat [inaudible] in kindergarten were from low income families. Remember, you start slow, you begin slow. We've got to make a change, and that change is how we approach it. So here's the longitudinal study. Here and a colleague following kindergartens with kindergartens and 10th grade. So so far you're with me, right? That's a longitudinal study. We assess them in kindergarten and we're watching their growth pattern [inaudible]. What he

found was beginning first wave that they understood their numbers and number line, specifically, they'll do better later on. Does that make sense? It's a number line beginning, which means kindergarten assessment should be assessed on a number line. Okay. Moms and dads, you are -- we're all taking note of this, right? They study reinforces the idea that knowledge is [inaudible]. And if they don't have a good foundation, they've got nothing to build on. It's the tower of Pisa. You can put a ramjet underneath it, build it, make it go straight back. The problem is it's still weak. This is the intervention, guys. So I will describe interventions. I will show you Tier 2 and Tier 3 interventions today. But I got to make sure you understand up front. If we're discussing Tier 2 and Tier 3 interventions, we got bad news. Don't look at this as this is an internal discussion moms and dads, not with your -- hope that mom and dad come in and go. "Oh, good. So you're going to do an intervention for my child. Good. We'll be back up." Please know the long -- the longitudinal effects are not [inaudible] get that strong instruction up front. If the intervention -- if you know -- listen, if you know in first grade that double digit addition is going to be a problem, stop waiting for intervention and put more time in the first grade, make it go longer in first grade, put extra time that would go towards double digit addition. That's the goal. All right. Research has also found -- here's back to that number line, that students who knew the basic math facts did better than others. If you have a curriculum that does not do fluency in the math facts, you are -- you are setting up for difficulty down the road. Okay? Now, that for some reason is still controversial in our nation. It isn't to Asian nations, it isn't to Russia, it isn't to Europe, but in the United States say you need to be fluent in your facts. Somehow, that's still controversial in this continent and our scores show it. Okay? So we got to get past that. I'm not saying you ignore concept. Your -- you must understand concept. They must understand properties, but they must also memorize. So there are techniques to do that. There's incremental rehearsal, there's spaced learning over time. There's things we need to talk. So here's the details. I'll show you an assessment before I show you guys theories. So some of my colleagues who came up with an assessment in a -- in a different state that might be a little big. I don't know if you can down -- oh, okay. And here's some of the things we came up with. I -- we need more number line here. We got some things we need to put here. But we wanted an assessment that had -- for an early five-year-old that wouldn't take much time, Right? We need a 10-minute assessment. So one was identifying the number of objects here. What's the new rule that goes with it? Some students will do what?

AUDIENCE MEMBER: Count.

AUDIENCE MEMBER: Count.

AUDIENCE MEMBER: Count.

DR. BRADLEY WITZEL: Count.

AUDIENCE MEMBER: Count.

DR. BRADLEY WITZEL: What will a student who has a different -- is beginning on a different track, what will that student do?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Good. It might be guess or they might figure out a subitization towards it. They might do small groups like, "I see four and three more. I see a group of four, a group of three." And then they might have to do four, five, six, seven. You'll have to count on for it. And you will see students do that. I got video. You'll see children do that. We go to do it for them and I'll explain how we messed it up for them? Children need to come in with a chance to subitize. We're the ones who say, "Are you sure?" "Try it again." when they get wrong rather than working off the groupings or the cardinal number there. The next one through -- oh, draw the number of objects. We just do that to get the kid active. So when he draws the number of objects, some of that by the way is attention and memory because as he's drawing, we'll he'll lose track of how many. You -- if you watch it and he draws two and he goes, "What's the number again? Three." We have an attention memory and comparison issue that we need to lock into. Here comes our open-ended number lines. We put some increments in there. This is approximately where are these numerals on this number line. All right. And we don't -- we know it's part of the measurement tape on when try to figure this out. It's approximations. This one is circle the quantity that's the biggest. One is pictures of objects which side or which numeral. And then our last one's here, this is kind of for the advanced ones to see, then they figure out addition, subtraction. Early kindergartens, you'll be surprised, a number of kids will come into them and -- did -- that tells you what mom and dad have done more so than what they've just developed here. And guys, I'm not trying to make money off this. The reason why we put this together was here's the research. Instead of waiting for the next numeracy to come out, and Nancy Jordan's going to come out with an amazing one, University of Delaware, Nancy Jordan. She's going to come out with a great number sense assessment. We wanted a quick screen here. But I hope that this isn't a surprise because I want you to watch this one little young girl. Thank you. Hi, this is Dr. Brad Witzel working with a kindergarten friend of mine named Caroline. And she said she wants to help us with addition. Which one do you want to do, Caroline?

CAROLINE: I'm going to pick this.

DR. BRADLEY WITZEL: Oh it's a hard one. Great. Show us.

CAROLINE: I ate six nuts on Tuesday and another three nuts on Wednesday, how many nuts did I eat over those, the two days?

DR. BRADLEY WITZEL: All right. Show us how you work this out?

CAROLINE: Well...

DR. BRADLEY WITZEL: Okay. Tell me about the student. It's her second week in kindergarten. So tell me about the factors here.

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Okay. Okay.

AUDIENCE MEMBER: She has [inaudible]

CAROLINE: She has had some serious background. I--this is my child when we went from University of Florida to Winthrop University. Winthrop University is the oldest lab school in the United States. That's why we went there. So we got the experience from this lab school. And I got to tell you the lab school did not like her techniques. And so we told them to don't touch her math, don't talk to her about math. We will do that. In other words, I'm not saying that they're bad. I'm just saying that I didn't want her to hear interference of data or interference of instruction. No, no, no. I want her to do it this way. So her reading was fantastic, thank you, miss, because I am not a good reader. So thankfully that someone in her family is. Still [inaudible] what I want you to do is watch her finger use for the numbers, like what six plus -- just watch her -- watch her finger because obviously a child who's not have been struggling times in math. Here we go.

CAROLINE: I have five in my hand and then I take one from the other hand and I put -- and I put three on this hand.

DR. BRADLEY WITZEL: So -- all right. Okay. Okay. Okay. I'll go right back to here because I'm -- Jane told me it's quite going to fun for just [inaudible]. All right. So what did she say? How did she [inaudible] in six?

AUDIENCE: Five in one hand.

DR. BRADLEY WITZEL: She's really got that subitization. She -- how many of our children are still counting and they hear six and they're going--making sure they got five fingers. It's still five in my hand. I mean, "I know the [inaudible] Mr. Witzel. I didn't know if I lost one along the way." And I had -- my friend in high school, he's named [inaudible], "He had two thumbs on one hand." He was basing eleven. His whole life -- does that make you look cool? And he would hold [inaudible] I got six. It was really funny. I thought at first it was really cool actually. So she holds up five and I have one more, it makes six. Remember, I mentioned in early addition before entering school? Here's where that comes. Then she says three more. Well, how did she do that?

AUDIENCE MEMBER: She [inaudible]

DR. BRADLEY WITZEL: She pops up three fingers. She can see that sometimes, she can -- she sees the cardinal numbers. That makes sense. She's got the cardinals in there. Here's the subitization, three. But she can't go beyond six in this. Watch as she puts up five and one more than three. And I gave her time. I said, "How many altogether?" Watch her lips and her eyes and tell me what she's doing. All right. Oops, I tracked it.

CAROLINE: And I put--and I put three this hand.

DR. BRADLEY WITZEL: So how many do you have total?

CAROLINE: I have nine.

DR. BRADLEY WITZEL: What did she do? Did you see her lips?

AUDIENCE MEMBER: Uh-hmm.

AUDIENCE MEMBER: What did she do?

DR. BRADLEY WITZEL: So how many do you have total? As far as she could.

CAROLINE: I have -- and I put and--I put three from this hand...

DR. BRADLEY WITZEL: No matter what I'm seeing now. So how many do you have total?

CAROLINE: I have...

DR. BRADLEY WITZEL: She's counting from one. Even though she knew that was five and knew that made six and knows that three more -- she knows that she has ten fingers because later on it is -- there's no reason to get through and make of too much [inaudible]. Well, I have ten fingers, so if I have one left - - she starts rationalizing how it works. But the truth is when I put on pressure she was counting as fast as she could by the time I finished my talk. And later on at school you'll hear that she'll say, "I didn't count it." She thinks that counting's bad. Not strategic counting. She thinks all counting's bad at some point. She thinks that she should be like Rain Man like two hundred forty-six, three clips. That's what she thinks math is. So we're trying to move on now. I love her theory, but that's it. And so anyway -- we -- I'm rude and I ask how did you know.

CAROLINE: Nine. And then -- so I would say six...

DR. BRADLEY WITZEL: Just bear with me.

CAROLINE: ...plus three is nine.

DR. BRADLEY WITZEL: Great. How did you know it was nine when you held up all nine fingers?

CAROLINE: When I have my fingers I know that I have ten fingers, but I have to use both of them for ten.

DR. BRADLEY WITZEL: Good.

CAROLINE: Thing's that I have five on each hand because that would it make more separately.

DR. BRADLEY WITZEL: More separately. Yes, I see.

CAROLINE: It looks...

DR. BRADLEY WITZEL: All right. It goes on, like, after a while. She can't say I'm cardinalizing. I'm subitizing. She's trying to avoid the word I counted.

AUDIENCE: I counted.

DR. BRADLEY WITZEL: So let's switch back to the [inaudible] of what do children do when they're introduced to math and they think they understand what they're supposed to do. So I got my [inaudible] colors here. Sorry. Sorry. Sorry for some of you, not for me. All right. But if I get a four plus three now with -- oh, by the way I should've shown that. When I do four plus three. See if you can tell whether [inaudible] "When I do four plus three." All right. When I'm going through this, I can just grab -- I know a three up. I see there's three dots, pull three. Watch a child do it. They're one, two, and three. So when they're adding up four plus three -- I want you to add four plus three. Here's some blocks, we can for. Four plus one, three. Remember, they not -- the memory is not there of three. One, two, three. And add one, two, three, four, five, six, seven. That's seven. Is that correct?

AUDIENCE MEMBER: Yes.

DR. BRADLEY WITZEL: What's correct about it?

AUDIENCE: The answer.

DR. BRADLEY WITZEL: The answer, but is that what's the most important in this early addition? Listen, most of our students by the time they can [inaudible] they can add. The problem is sometimes -- the problem is it's the approach that we're taking with it. Let's go a little faster here. Look for four. Look for three. If the child messes up and grabs four here, do not say, "Oh, you better check." because the child only has one strategy and it is...

AUDIENCE: Count.

DR. BRADLEY WITZEL: ...count. Which means you just said, "Don't work at cardinalization, work at counting." So we've got to say if he does that, you go, "Oh, that's four, not three. That's three. Do it again. That's three. Good." The correction is an instant, not a count correction. So here comes the counting piece. Grab the largest group, count on and say, four. I covered up, four, five, six, seven. So far how many of us do that in that way? Or if you don't teach that age, that's--that makes sense. You got it. And it makes sense, but it has to be taught. You can't just hand manipulate the children and say, "I'll help--this will help you." Because what they're going to do is they're going to count from one and we're going to count -- and you're introducing wrong procedures.

REBECCA: You're right, it has to start from [inaudible] start on [inaudible] great and you...

DR. BRADLEY WITZEL: That's correct.

REBECCA: ...hang on, it makes it so hard.

DR. BRADLEY WITZEL: If you didn't hear Rebecca, I will repeat that. I want you [inaudible] videos [inaudible] what Rebecca is saying in fourth grade there's still not counting on. They're working on trying to figure out the large adding first then the small adding and then they're counting again. Correct? And they're still counting incorrect because counting should start with the number...

AUDIENCE: Zero.

DR. BRADLEY WITZEL: ...zero. One is a distance. And as I joke, I learned that playing board game. I play board monopoly with my children. Not that kind of board. They were excited, bored on my end, but board Monopoly. And they roll a six--I think I do--I got a dice in there. When they roll a six and they go, "One, two, three, four, five, six." And so they're grouping only five and I'm trying explain and she goes, "But I count it six times." Then I said, "Well, actually six has seven numbers in it." And, I said, "Oh, my gosh. What am I doing? Stop, stop, stop. Six has seven spaces. And you pull out the number line-- actually, I just pulled out a -- just a yard stick and I showed her with a -- because yard sticks -- don't get the wooden one's get those metal ones because it's a porcelain enamel of [inaudible] so you show them. Let's count the number over. One, two, three, four, five, six, that's [inaudible] there. She goes, "But I had right." Okay, that's fine. You started at one, two, three, four, five. Oops, six, but what number we have?" She goes, "That says five." "Yeah." "Oh, my gosh, it's the space." The child has to go through it. If that doesn't work, my suggestion is those of you who will teach full body movement children, have a measuring tape on the ground or just Scotch Tape -- mask -- Scotch Tapes are messy, masking tape and have them walk through it. And those of you, my middle school teachers, not only is it positive, it is negative and you got to go through it. Help them understand of -- for you guys facing the positive direction, negative one and then if I'm subtracting a negative one. That's why it goes positive. So you got to -- you got to start teaching this directionality. Listen, everything up to algebra is trying to get them to a one dimensional understanding [inaudible] through algebra and geometry we'll get to a two and then eventually a three. If they get really excited, they'll hit the fourth dimension one day, but Stephen Hawking's isn't here, so we're just going to have stay at one. How do you [inaudible] Hawkin's [inaudible]. All right. All right. Number sense can predict -- another one is we can predict by that assessment that I did earlier. You can predict six years down the way of how students are doing, which means we got to catch it early on these. Here's the big five, word problems is actually the language and word problems. Number and numeral, magnitude comparisons, how they count, fact fluency, and then the language of math. And we're done with word problems, but the truth is it's the language of Math. If you say, "Well, where's problem solving?" I love that one. "Where's problem solving?" It's not in math. I don't know how to make that claim, but everybody goes, "Why do we have to study math at all?" It's for problem solving. I probably figured out the subject that you don't teach problem solving. Right? Why not wait until math to teach problem solving? Shouldn't it be how they get on the bus? Shouldn't it be how I hit the class the most efficient way? I got high school, I got kids who didn't learn problem solving here from one room in a building to another room in a building. I mean, problem solving has reinvented life. It's not a math discussion. It's an everyday discussion. So the math language is what's coming out. All right. Number sense has been defined several different ways. National Math [inaudible] find it in a -- in an assessment way; small quantities, computation and magnitude. What we're going to do is let's go through some activities that show it off. I can't help myself. I love research. Look at that. Okay. All right. Is anybody doing -- anybody know these sayings? Around the tree and around the tree, that's the way we...

AUDIENCE: We make a three.

DR. BRADLEY WITZEL: ...make a three. Down and over and down some more, that's the way we make four. Five is weird. Fat old five sits down and around, pop a hat on top. Seven, you have to be careful. We're using seven, because seven is across the sky and down from heaven. So I'm trying to find a non-South Carolinian way of doing that one. But again each one of this has a way approaching them. We've -- this is a sing, it's a word choice. How do you all teach children to write numerals? What you all do with them? Tell me some of the cool techniques you do with kinder children?

AUDIENCE MEMBER: [inaudible]

DR. BRADLEY WITZEL: Shaving cream. I know you said shaving cream.

AUDIENCE MEMBER: Sand.

DR. BRADLEY WITZEL: Shaving cream, sand. The [inaudible] sugar and whipped cream. And I taught in Florida so we have lots of visitors through the night. But, yeah, shaving cream is good with menthol. Some of your kids will really get that. I'm not kidding you. That's just what happens and enjoy it. Okay? So have fun. Don't take pictures and tell mom, but it's the things that are tactile, sandpaper, old chalkboards, whatever that is that lesson go down. What things made you creep out, it might not make them creep out. It might be exactly what they need to trace over with. If you ever seen, I mean there's programs out there -- [inaudible] programs. There's programs out there that actually if you lock them into the size of a board, most numbers either start top left or top middle. One is here, two -- oops, top right, two is here, three is here, four on the side, five, right, six. I have it here, and I have it here, seven, eight. No matter where if you see stars, the numbers are easier than lower case letters, it's like capital letters. They start on top left or middle. And you have them in a fixed, confined space so the kid doesn't go all over the place. My wife's [inaudible] childhood and my gosh, the number of things that she has shown me that we do to help these kids get through that. But one, they surely understand the numerals. Is there anybody doing that presentation? I mean, I know this is pragmatical -- touch math, but the original research, documentation, a couple of nods. Tell me some pros about documentation math. Don't be hesitant. Yes.

AUDIENCE MEMBER: Like counting on their hands, it's not as obvious they're doing it.

DR. BRADLEY WITZEL: Good. It -- it's a transition from a concrete -- another lesson, from a concrete to a [inaudible] so it transfers out so they're stop -- they won't have to count their fingers. Another positive here is if you look at the original Arabic, it's the number of angles, interior angles of each numeral for all of us. And there might through -- their seven, very different. It's a seven with all sorts of little doohickeys on and all. But it's -- it actually is the number of interior angles in the number. So -- I mean it has an original base. It's -- in the marketed program, I know that you're thinking that, you're going, "But it doesn't -- that's a marketed program." I'm giving you the original research. The original research also with their [inaudible]

talks about -- he's from University Quebec at Montreal, he also says they have not [inaudible] effectiveness of students with high incidence disabilities. The effectiveness they have found so far serves the low incidence because they're still trying to count. Because with Brad, you're trying to get them to count from zero, I'm just trying to get them to count. Okay? And that's where some of the effectiveness you'll see in there. So you think they're adding, you think they're multiplying, what they're really doing is trying to count. All right. Cardinality, it's the groups, right? You see five babies, you know it's five. If you see three bowling balls which have three adults in them which else, right, you know that's three. But the idea is to make sure that you can go through this. So I'm going to check your cardinality on this. Can you see the number? How many of you are like Rain Man? So we're going to go through -- do ten frames. Ten frames are great. They show their size and quantity where either a three plus four, five plus two and it shares that with us. But I want to see what you do. Let's make sure that one -- all right. All right. I'm going -- I'm going flash some numbers up here for you. And I want to see how many you can see and instantly recognize. Ready? On your mark, get set go.

AUDIENCE: Five.

DR. BRADLEY WITZEL: Five. How did you know it was five?

AUDIENCE: There's a one missing...

DR. BRADLEY WITZEL: You saw -- you saw one missing here, but it was replaced down here? So your eyes -- those who saw it that way, your eyes are scanning from left to right, top to bottom. Welcome my fluent readers. How many people saw two and then three more? That's me. I'm a graph lover. I saw two in the left column and then three more through and over there. Anybody else see it a different way? How did you see it?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Four and one? Which means you immediately went to the addition. All right. What you -- my daughter did, where she went five and one more. That's six. She immediately [inaudible] addition.

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Oh, you saw the three and two? Oh, you grouped at triangle. Wow, that's new. Okay. My eyes don't do that. Yours do, which is cool. All right. Ready? Another one. Mark, set, go.

AUDIENCE: Five.

DR. BRADLEY WITZEL: Five? Gosh, I should choose a new number, but you know -- you know I was doing it. How many people saw three and then two more? You got it? A left to right. How many people saw two, two and the next [inaudible] on there? You got it? So, I mean, if I -- we have that graph piece. You -- you're using like page scanners and all that. I'm not here to classify people, but we do have our

scanning material. Who else saw it a different way? You saw a W? There's five points in a W or something like that.

AUDIENCE MEMBER: [inaudible]

DR. BRADLEY WITZEL: You just know there's five points something W? What else?

AUDIENCE MEMBER: [inaudible]

DR. BRADLEY WITZEL: I have not. So in other words her mind, she's able to -- talk about the geometry first. She's able to instantly move things in her eyes to know the things would fit, which by the way is the basis of high school geometry of that predictive analysis of where it will fit. So what I'm getting at is each one of these guys is a way of seeing if we can cardinalize because our children should as well. Can I have one more? Of course, as achievers. I put 10 on the bottom just to mess with you. But tell me what the number is. It's?

AUDIENCE: Twelve.

DR. BRADLEY WITZEL: And I'm going to tell you, the number is one, ten, and...

AUDIENCE MEMBER: Two.

DR. BRADLEY WITZEL: ...two more. You can't say and. Yes you can. And means a decimal or addition. So if someone says -- you think about the contest and they go, "The number's one thousand and two," do they mean one thousand plus two? If they meant 1002, that's incorrect, but you have to find out what they meant by it. Now, an owl worker says the hurricane is coming at one hundred and forty-three miles per hour. It's obvious. He's not talking about two hurricanes. Okay? He's really just messing up the number, but that's where we need to go. By the way if you've seen [inaudible] from [inaudible] to how [inaudible] and he said, about a decade ago and said, "Please say your numbers correct. This is embarrassing." Isn't that funny? And he apologized the next day. So we need to change that perception of how to say numbers in the language as well. But tell me what can you do to teach cardinality in content? So pick yourself just one or two ways. Turn to your neighbor. Talk to your hand, whatever is necessary. But think one or two ways that we could help infuse what cardinality -- count -- proper counting in our curriculum because there is no curriculum that says start here, right? It doesn't know we can't. Give you a minute. I'm sorry?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Okay. It's okay. [inaudible] website. [inaudible] website. It should be downloaded. And, yes, and the assessment as well.

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: What's up?

AUDIENCE: [inaudible]

DR. BRADLEY WIZEL: Take your time, the website [inaudible] you can download it.

AUDIENCE: [inaudible]

DR. BRADLEY: And -- yes, and the [inaudible] all right. Wait. That was good talk. Look at that. Love it. All right. I want to show [inaudible] 600 people [inaudible] as well, [inaudible] it's not everytime, nobody would speak, so tell me [inaudible] groups. What's wrong when we confuse better or more, counting in cardinality exercises into our -- and I'm not talking about the childhood classes across the board. What can we do? You didn't just talk about lunch. Come on, what did you talk about this? What can we do? Give me some techniques. Yes?

AUDIENCE MEMBER: Manipulatives.

DR. BRADLEY WITZEL: Manipulatives. And not just hand them manipulatives but teach them what to do with those manipulatives.

AUDIENCE: Right.

DR. BRADLEY WITZEL: You got it. Now, most manipulative companies will not tell you to teach the manipulative because we have a philosophy so in our country. We're one of the last socially constructed Math countries. That's what happened. Most people say, "No, you better teach the procedure," but those are the high-performing countries. Not all countries say that. So, one, manipulative as a teacher. What else?

MICHAEL: [inaudible]

DR. BRADLEY WITZEL: You got it.

MICHAEL: I'm even adding -- separating two different colors.

DR. BRADLEY WITZEL: That's a great idea.

MICHAEL: [inaudible] the idea of most people [inaudible] numbers.

DR. BRADLEY WITZEL: Tell me your name. I'm sorry.

MICHAEL: Oh, Michael.

DR. BRADLEY WITZEL: Mike. If you didn't hear what Mike said, he goes the ten frames and then those dot pictures to go with it, and what he saw in that is, a couple slides ago, the -- is the multiple colors on this, so you can see which added is where. And by the way, you might see that my pattern doesn't match everybody's. Some people like three plus four in the bottom. Most of us go left to right, top to bottom, and if you're really good in Math, Algebra, all of your math will be left to right, top to bottom. So, in that -- in that case, I do try to go left to right, top to bottom early even in my ten frames. I just want [inaudible] I

know you're thinking, "I thought Math was right to left." Not if you don't understand place value. If you understand place value, it's always left to right, top to bottom. Oop, [inaudible] I'm trying to set up my own next step. What's the next one? Give me some more ideas here. Yes?

AUDIENCE MEMBER: Spread [inaudible] more of your activities throughout the day...

DR. BRADLEY WITZEL: Love it.

AUDIENCE MEMBER: ...so [inaudible]

DR. BRADLEY WITZEL: I was working with -- again, I honestly have the privilege -- and I don't mean that to be cute. I have the privilege working with probably the 30 districts in the nation. And I'm so happy about it because of the techniques. One of them, we just had to sit down in a smaller district. We got every elementary teacher in the room -- or in the -- in the schools to come together. And we talk about how many hours a day they would teach Math. And so a first grade group said -- luckily, one first grade group went an hour, an hour, an hour, an hour." And one teacher goes, "Oh, no. See, I don't have time for that. I got literacy and social. I do half hour a day." The second grade team immediately stood -- one teacher goes, "I know it. It's your kids." Now, you think that's negative and -- maybe I'm getting older and goofier. I'm okay now with people. I have confidence now if someone challenges me on my own work. You go, "Oh, I need to rethink that." Maybe it's parenthood that taught me that, maybe it's marriage that taught me that, but I'm okay with the critique that obviously, in a -- an attempt at positive reaction. There's a way to do that, of course, but from then on, his goal was to do an hour a day but he says they can't attend to it an hour a day. So an answer was they don't have to attend it for an hour. Why don't you go 10 minutes here, 15 minutes here, 20 minutes here, just split it up. And of course, he goes, "Oh, yeah." There isn't a Math time. There's no Math -- well, it's Math hour [makes noise] no, no, no. The bell doesn't go off and, oh, stop everything in Math. I'm waiting for that time. Stop everything in reading, I'm waiting for stop everything in computing. Multiple -- all right. Give me some other ones here, other techniques. Going once, going twice? All right. I need you to hear a little girl. I apologize to those who know this young lady. She hates the video and I love the video, well, because it's embarrassing. Not right, where are you? What she's going to do is I'm going to ask her to do a subtraction problem but it's not the subtraction problem that's going to shock you on how she approaches it. I want you to focus on the numbers that she uses with it.

[VIDEO STARTS]

BR. BRADLEY WITZEL: Hi. This is Dr. Brad Witzel. We are going to talk to Laura Witzel, a local fifth grader, about subtraction. She's going to help us out. She has a new way to do it. Let's do one first, Laura. Can you try 82-17? And tell us what you're thinking the whole time, that way, we understand your thoughts.

LAURA WITZEL: Okay. Eight tens minus one ten, that would be seven tens that I'm just going to write as a seventy. Two minus seven is negative five, so I'm going to write minus and then five. And that would be 65...

DR. BRADLEY WITZEL: Nice job, Laura.

LAURA WITZEL: The answer is 65.

[VIDEO ENDS]

DR. BRADLEY WITZEL: All right. She was doing Math differently than her teacher -- or I think it's like second grade teacher. Now, she was doing it different than her teacher understood. And her teacher knew me and she goes, "She's [inaudible] I probably shouldn't correct it. I probably should just ask." So I said -- so she said, "Would you -- could you make a video of what's happening because in a large class, she wasn't verbalizing. She knew that she was doing it differently and she -- and she's very [inaudible] [makes noise] could you tell what you're doing? "Yeah, privately," because dad always said, "Don't mess with her Math. Don't mess with her Math. No, no. No, stop, don't talk to her." So in this regard, she's doing it differently. What did she do? What if she sound different, say, in doing this computation?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Uh-hmm, she starts place value. Eight tens minus one tens, that would be seven tens -- this what she say. She's used to us...

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: To save you poor teachers, I'm just going to write it as a 70. You don't think that she's [inaudible] she's run aground on this. In classes, they told her she's wrong on [inaudible] and we'll get to that. Then she said -- and I know you're going negative numbers on [inaudible] she said two minus seven is?

AUDIENCE: Negative five.

DR. BRADLEY WITZEL: You know how she's doing that? They [inaudible] this about second, third grade. Do you know what she's doing now in sixth grade?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: And she's so frustrated because she goes, "I'm sitting here and I'm in middle school and I'm in gifted classes and I am bored to tears. I'm so irritated. I don't want to go back to school. Can I just be home schooled?" I'm, like, "What are you talking about?" And by the way, with the highest performing district in South Carolina. She was, "I'm just so irritated." I go, "Why?" "We're doing integers." I said, "Where did you learn integers, honey?" "When I was six." So, she's a little [inaudible] other kids, like, "How did you get it, Laura?" She goes, "It's a number line, go left." That's what -- that's

what she's saying. And so her teacher's, like, "How did you do [inaudible] her teacher's awesome, too. Her teacher goes, "Yeah, I don't know what to do for you right now." So, she starts giving her another work to do. Laura's, like, "Oh, could I not be first grade?" Because internationally, when you're on a number -- when you work in Celsius internationally, your whole winter might be -- all her winter here would be a negative nine. Right? At least the last three weeks would be. It's just what degree of negative we're talking about. Oh, my gosh, positive one, woo. I [inaudible] kids -- I mean, whatever it is. So in that regard, she looks at a number line as a very clear, understanding visually -- mental visual of a number line. Remember Gary's work? It predicts later achievement. Well, here we are and she's bored. She goes, like, "I [inaudible] classes guys." This is a good -- with a good memory but if I ask her, "Are you gifted?" Her answer was, every time, "Not really. I got kids in my class much smarter than me. I've just been taught." Higher expectations, because right now, the expectations we put on our children is what has been put on us and we got to switch that up. Oh, by the way, she will tell me I'm wrong because she loves her teacher and her dad's -- if you're closer to home, then [inaudible] okay? So, she's about to correct me.

[VIDEO BEGINS]

LAURA WITZEL: And in other ways, it would -- you just cross this out.

DR. BRADLEY WITZEL: Listen to what she says.

LAURA WITZEL: Seven is...

DR. BRADLEY WITZEL: Listen.

LAURA WITZEL: ...and that's twelve. Twelve minus seven is five, seven minus one is six.

[VIDEO ENDS]

DR. BRADLEY WITZEL: Hold on, hold on. She corrected me with an incorrect approach, right? How did she correct me? Look at the face. I love her. She hates this, by the way. In Massachusetts, puts this up and shows it. I'll be having a discussion of it and my daughter's irritated by it but it's okay. She's in middle school. [inaudible] I'll get her again if I could. She says twelve minus seven is?

AUDIENCE: Five.

DR. BRADLEY WITZEL: Five. Good so far. Seven minus one is?

AUDIENCE: Six.

DR. BRADLEY WITZEL: See, that's incorrect. Seven minus one is six, of course, it's correct but that's not what she wrote.

AUDIENCE MEMBER: Seven tens...

DR. BRADLEY WITZEL: That's seven tens minus one ten is six tens. If she had even said 70 minus 10 is 60, I take it but she decided to attack me back with an incorrect procedure. So I said it to her teacher and her teacher laughed and said, "I love that she corrected you." Incorrect. And then she wrote to me again and goes, "Did I do that?" And I didn't answer a word. I just said, "I love you." She's one of my former grad students. I said, "I love you," and she goes, "Oh, could you send me another video?" And again, look at that face. Love it. That -- she's middle school now, so I see that everyday. All right. So don't think I'm getting negative I love her -- yes, ma'am?

AUDIENCE MEMBER: Laura said that she -- in the video, she said, "I'll write down [inaudible]"

DR. BRADLEY WITZEL: She would have just written seven in that space and then went minus five or even then, the calculations in her head -- on a number line, she's going up to seventy and five less. That's all she was doing mentally, so she [inaudible]

AUDIENCE MEMBER: [inaudible] she wasn't necessarily right then. She [inaudible]

DR. BRADLEY WITZEL: You got it. So let's talk about students at the intervention level because you might be thinking I'm describing this as some gifted thing and, boy, I'm [inaudible] he doesn't know who he's talking to. This is an intervention. Why is that an intervention what you just saw? You think it's a gift then and I'm going to tell you no, it's an intervention. Let me -- let me show you what we do with calculations. Maybe that will make it easier. I didn't bring my tissues. Oh, so, I did, I did. So, if I do 123-45, how do I do this with our students in our classes right now?

AUDIENCE MEMBER: Borrow.

AUDIENCE: Borrow.

DR. BRADLEY WITZEL: I borrow because you start on the right, of course. And I say you can't take five from three which you can, right? And so I scratch this out and that becomes a one and then another one shows up over here, right? Okay. So then I go 13-5 is?

AUDIENCE: Eight.

DR. BRADLEY WITZEL: Eight. And then -- well, that's a one there and nobody thinks that this is actually eleven tens. They actually group this out and put it over here. Then I have no idea what's going on there. You got to love poor children for having to follow us. And in this cell, 11-4 is...

AUDIENCE MEMBER: [inaudible]

DR. BRADLEY WITZEL: Is that incorrect? The answer is correct but what's wrong with this whole approach?

AUDIENCE: The place value is messed up.

DR. BRADLEY WITZEL: Place value is messed up. The language is messed up. Nothing is from right to left. And I got to be honest, I don't know if you teach different children I do but it's not that their Math is wrong. It's like the organization is a disaster. And next thing I know, I'm turning paper sideways to right [inaudible] lines can work. Well, let's do it this way. So, we can just follow the tract. What I'm doing is I'm trying to force an Algorithm on it that really isn't helpful. But when you force feed that Algorithm, you're going to cause more trouble. So, I keep using that same marker, I don't have it [inaudible] so when I get 123-45, take a look what I'm doing here. Twelve tens minus four tens is...

AUDIENCE: Eight tens.

DR. BRADLEY WITZEL: ...eight tens. That's what she was going to do. And then it's three ones minus five is minus two. What's two less than eighty? Seventy-eight. The notation is much different but there's a difference here where some of our standards go through. And our standards, they -- well, they shouldn't understand negative numbers until sixth grade. And then we're going to assess that this child is [inaudible] I better wait until sixth grade. You understand what I just set up? I -- what am I going to do in sixth grade or seventh grade? What I'm going to do? Oh, by the way, welcome to sixth grade. You know, everything we learned has been on one side of the number line. It's a number line, not a number ray. We've been tricking you for all your career. The numbers go the same that way. What? What we have to do -- anybody have to do number lines [inaudible] the new kindergarten number lines, anybody have them? They go from negative 20 to positive 100. And the point is to make sure that we understand negative is a direction. It's a subtraction. When should you introduce negative numbers to children who struggle? When you introduce subtraction because it's a number line moving positive and negative. Questions?

AUDIENCE MEMBER: This is -- this is great. I work [inaudible]

DR. BRADLEY WITZEL: I love [inaudible] I [inaudible]

AUDIENCE MEMBER: [inaudible] subtraction is that everything else [inaudible] and you say they'll have to write [inaudible] I mean, like, because there's a lot of [inaudible] they do really love it and now, here's this huge shift [inaudible] and that does at least [inaudible] subtraction [inaudible]

DR. BRADLEY WITZEL: And they get really good and understand negative numbers after that sixth grade standard, then they're going to get it again, but we don't even teach them yet. [inaudible] of teaching it? When we get to right terms then we start getting back to that left -- that left to right, now it's a negative. So they don't even see it until then so we have a gap. We have a [inaudible] gap and by the way, when -- even Common Core came out and did this [inaudible] work with several publishers, they immediately called and said, "Wow, let's start getting this some negative interventions out there." In other words, they saw the error's about to happen. And we're going to call these kids [inaudible] because behind it all and the truth of the matter is it's a curriculum error. It's a standards error. So, here's the problem. Wait, that's not in my standards. Listen, that's the craziest thing, the time and money, you're

going to wait until second grade to introduce him to a clock, right? Because understand as I work with one school district and they go, "No, no, no, we're not allowed to that until second grade." No, no, no, guys, guys, Common Core are benchmarks. They are not standards. The original standard when they were written was -- these are the benchmark areas. But then teachers started teaching things differently so it got really confusing. And so what they did, they called them -- they kept them called standards but then it was [inaudible] objectives, yearly, annual objectives. Well, we're kind of back to the beginning of -- here's some broad -- here's some broad standards. These are all benchmarks in there. So, it means by the end of the second grade, by the end of the sixth grade. That doesn't mean you started sixth grade. That means by the end of sixth grade, they have mastered and I hope that changes your structure of what you think of Common Core. Common Core is not designed to set up your curriculum. Common Core is designed to set up your assessments. Makes sense?

AUDIENCE MEMBER: Just a comment.

DR. BRADLEY WITZEL: Yes, ma'am?

AUDIENCE MEMBER: I've got some kids with learning disabilities and we're going through the [inaudible] right now.

DR. BRADLEY WITZEL: Timing, right. Yes.

AUDIENCE MEMBER: This is awesome because their issue is far [inaudible] right now with regrouping is they need to go between subtraction and addition. And they have several problems saying this is a subtraction problem, so then we have to borrow 10 and add it [inaudible] to the three to make it 13 and where does that 13 go because you're bringing it over into the ones column but you're not really bringing it to the ones column [inaudible] floating up there in the air.

DR. BRADLEY WITZEL: It was always there.

AUDIENCE MEMBER: But in their mind, they can't visualize this whole process.

DR. BRADLEY WITZEL: So if this negative by the way is far out of reach, the next step to doing this would be -- I got to go [inaudible] -- would mean -- make sure that the child understands what numbers make up 123 and what numbers make up 45. Now, watch what I do, everybody wants to put the four tens over here. Why can't I put four tens over there and five under that? Because now, I got ten tens minus four tens, that's sixty. And twenty minus five is fifteen plus three. What I'm trying to do -- oh, that big icon is there. Well, I want to make sure you see is that we are so stuck on our one approach, we kind of lose track of what subtraction's even asking us to do.

AUDIENCE MEMBER: AUDIENCE: Right [inaudible]

DR. BRADLEY WITZEL: So in this case...

AUDIENCE MEMBER: ...they're always taught [inaudible] use their number line for everything [inaudible] so much simpler [inaudible]

DR. BRADLEY WITZEL: And a number line is not a bad approach but what I'd have to do here is I have to identify where one -- where twelve tens will be on this number line...

AUDIENCE MEMBER: Yeah.

DR. BRADLEY WITZEL: ...then go back to four tens.

AUDIENCE MEMBER: Right.

DR. BRADLEY WITZEL: And really help them understand it, so if you don't -- if you're catching on to the language patterns that I'm throwing at you right now. My daughter, the one you just got to see, failed her kindergarten [inaudible] remember when I said, "Don't touch it. Don't mess with it." She failed it. So she went to kindergarten. They did a kindergarten screening. "Yay, come on in," right? And they don't say, "You can't really fail it," but you can be placed in the lowest group. And they said she didn't know her numbers. And it's a good gesture, so they -- she got placed in a lower group because how did she say her numbers over here on the line? They said she knew her single digits, three and eight, but she couldn't do [inaudible]

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Because she would say one ten, three ones, three tens, one, and then she wouldn't even say the number one. She goes, "One, ten, three, three tens one, four tens five." And then they said, "Oh, she doesn't know her numbers." Yes, ma'am?

AUDIENCE MEMBER: The reason to that is when she was young, you introduced this number line [inaudible] she started zero when she counted...

DR. BRADLEY WITZEL: Yes. Yes.

AUDIENCE MEMBER: ...[inaudible] and then as you get her [inaudible] direction [inaudible]

DR. BRADLEY WITZEL: Oh, for the negatives?

AUDIENCE MEMBER: Yes.

DR. BRADLEY WITZEL: It was a situational thing.

AUDIENCE MEMBER: [inaudible]

DR. BRADLEY WITZEL: She was -- honestly, she was doing the garden. I know it sounds funny. She was digging at the garden and dad said, "I like getting dirty, too," so I'm digging in the garden with her. And she said, well -- I go, "How deep should we put it?" "I don't know." So we got a little scoop. "Well, let's go two scoops deep." And that was it. So she went two scoops deep. And she stacked and go

through the same area. I said, "Isn't it funny that you stacked two over here and there's negative two here?" I said, "What happens if I put them all back in?" She goes, "It's the same place." I go, "Let's call that zero ground, negative one, negative two." My wife is from Montreal, so guess where we go every winter. And of course [inaudible] "Oh, it is cold. It's negative 60 today." And I went, "There we go. Well, that might freeze your skin when you go walk outside." "Is that bad?" "Yes, that's bad." And then they get this discussion from that. And so, they -- I'm not getting -- from then on, it was a positive versus negative. How much do you owe me? How much do I owe you? Listen, our federal government knows you can't do this because [inaudible] 65 is less than 56, you subtract that. But if 65 is greater than 56, you subtract that. They work everything in absolute values because Americans can't get negative numbers. So why did we take a Japanese Math [inaudible] and then put negative numbers further on the line? Because they knew we'd fail. Rather than -- like in our textbooks, guys, I'm not kidding. In our textbooks, why do they skip -- how did you [inaudible] these tricks? That's a good option because if suddenly that challenges our Math, we're, like, "Oh, what is that? Next." Rather than, "Oh, so that's where this is coming from." That's the challenge we have. So when my daughter gets through this, we actually at home, a zero, one, two, three, four, five, six, seven, eight, nine, one ten, one ten one, one ten two, one ten three, one ten four, one ten five, one ten six, one ten seven, one ten eight, one ten nine, but I messed up, one ten ten. Ten ten, ten ten, that's two tens. Two tens one, two tens two, you get me? Question? Kristen? It's Kristen. I'm good. Kristen just said, "How do you know my name?" Feeling. Her name is Kristen? Kristen. Sorry [inaudible]

KRISTEN: [inaudible]

DR. BRADLEY WITZEL: Yes.

KRISTEN: And so I'm going to [inaudible]

DR. BRADLEY WITZEL: Excellent. Excellent.

KRISTEN: [inaudible] people say that [inaudible] is this [inaudible] you say of this number line [inaudible] thinking that your [inaudible]

DR. BRADLEY WITZEL: Okay. Now, Kristen is a [inaudible] so she asked, is the increments -- is the increments a problem with that? And I hate to go back and forth with this. The research comes from Holland. It's called open-ended number line or open number lines. And an open number line, a lot of different -- everything on an open number line, now, and I'm going to caution you, uh-uh, better have the increment set up segmented first so they know the jump pattern. Teach them that the patterns from one to two, right? They must be the same just like I mentioned. We'll be going one-fourth to two-fourths to three-fourths to four-fourths. So the patterns must be set ahead of time. That's where you got to teach the visual-spatial skills. And I think [inaudible] open-ended number line and underneath it, I'll put graph paper so that there's kind of, like, hue in the background and seeing that those segments are built in. So its partner [inaudible] is doing the open-ended number line and 123-45. My kid's [inaudible] their line,

trying to mark it up for them. Hey, let's make this zero, let's make this two and three, you're about here. Let's work from there. So in other words, some of our students, we have to be explicit in that instruction. Are those of our students? They have that spatial skill and at this point, our research is so weak. We think it just comes from [makes noise] knocked into your brain. Somebody zapped you with an electrical bolt. When the truth of the matter is somebody went on instructional to make you understand segmented in group sizes. Just like, for example, how did my daughter do it? When I get a scoop, all the scoops the exact same amounts, right? It's that precision that sometimes works.

KRISTEN: So, what would you recommend [inaudible]

DR. BRADLEY WITZEL: The -- Kristen asked, kids who have greater [inaudible] number sense, would you go back to spatial? I'd pull out a ruler and I'd start measuring [inaudible] and by the way, a yard stick is hard, a meter stick is perfect because decimeter is something you don't think about but decimeters, there's tenths of a meter, are so important for that child to see. And now, if you start putting around -- and you could -- we're not going to [inaudible] on the edge and all that. I'm not kidding. I had a -- I had a carpenter in one class who got irritated, called my [inaudible] love him. Thank you. One of those dads [inaudible] thank you. And so we make sure they measure a lot. And again, you're going back to [inaudible] all right. For this -- so my daughter's taken through these and she fails it. Anybody do students with IEPs? Not a lot of hands. I love student -- my first boss is Marcy [inaudible] Kovach. She invented student with IEPs. It's one of my reasons for switching to Special Ed. I love Marcy. So Marcy actually believed that if you know anything about a child with a disability, that child should be there. I know you're thinking some of the language, oh, my gosh, I would -- uh-uh, the language, it goes much better than [inaudible] No, I'm not a kid anymore. No one is belittling the child or say, "Well, he doesn't work hard," because the child's right there. Good [inaudible] it changes the meaning. So guess who's at my meeting? Well, they asked Laura to sit outside during this. So after they go through this, I said, "So my daughter doesn't know her numbers?" "Yeah, so we're wondering though. We're a month in this school and she's really caught up fast." I said, "Oh, okay." And of course, the principal knows me well and the teacher who knows me well, they go, "You want to tell us something -- by the way, I've got a bruise on my arm now because my wife was doing this. Tell him what you did. You screwed them up, tell him." All right. So I'm [inaudible] that. And I invited Laura and I said, "Laura, what's this number?" "Three." "What's this number?" "Eight." [inaudible] "What's this number?" "Dad, you want it their way or your way?" Now, Mike, the principal goes, "Oh, boy." "Why did we do that?" I said, "That's their way. They call it 13." She goes, "Then [inaudible] 31." She goes, "The students get -- do the children mess that up a lot?" "Yes," I said. She go -- and of course, the teacher goes, "What did your dad call it?" One ten three, three tens one, four tens five. Its sizes. She goes, the one -- the number on the left is ten times as big as the one on the right. It's just -- you can't write it that way, it'd be really weird-looking. Math on a flag wheel, thank you. Right when the teacher spoke -- teacher and principal, "Oh, we got it." So guess what I did? The next week, I'm back in the schools teaching what we're doing. I said, giving up

new standards and I don't understand that. Yeah, I know. By the way, Cardinality, there are eleven Cheerios on that stick. Why would I not correct it? She thinks there were...

AUDIENCE MEMBER: Ten.

DR. BRADLEY WITZEL: ...ten move along. I said, "How many -- how many are -- how many Cheerios did you put in each stick?" "Ten." "Perfect." I mean, why wouldn't I get in the middle of this? Actually, there's an 11th. I know it's really small. Really? She just told me she knows the understanding, move along. So in this case, I worked with Mahesh Sharma, the standards are very, very distinct that you must understand place value language as early as kindergarten. And aren't you surprised that they focused on the teens because the teens are where moms and dad's are really good at messing up. Not because they want to mess it up, because -- I still do it all the time. I mean, how many miles until we get there? At least twelve more miles. One ten, two ones. But see, my children think one ten, two ones first, twelve is the friendly version. Welcome to Korea. Okay. I'll stop. US. We are a literacy-based society, so when we do the A, B, C, D but those aren't the letter sentence, are they? We focus on [speaking in foreign language] right? Bless you. There are two different versions of our alphabet. Let's get to the rest of the world and the -- [inaudible] countries in math. There are two versions of their numbers. There's a singsong counting, zero, one, two, three, four, five. And then there's one, two, one ten one, one ten two. And the Koreans, they're not counting eleven, twelve, thirteen, their singsong count eleven, twelve, thirteen but when they see thirteen and someone says, "What's that number?" It's one ten three ones. Just like if someone looks at you and says, it's a P but what does it mean, right? Or my kids spell as [speaking in foreign language] that it means [speaking in foreign language] and so whatever that is, that I never got. It's a -- it's a [speaking in foreign language] I have a [speaking in foreign language] okay. But this Common Core works throughout and I just wanted you to see, that place value in kindergarten, it's all over First Grade and then if you were in our earlier session, it actually builds into the progressions of later learning. In other words, if it's missed in First Grade, and they didn't have it and you're a Fourth Grade teacher, and you're going to rate models, you're teaching place value. You may not want to teach place value, it might not be in your standards, but guys, they didn't get it by the end of First Grade. Welcome, your turn. Oh, I did this one earlier. I hate to repeat the same thing, but my daughter's been the place value, so those who heard it earlier, I apologize. How many hundreds are in this number? They want you to say what?

AUDIENCE: Two.

DR. BRADLEY WITZEL: Two. How many tens they want you to say?

AUDIENCE: Four.

DR. BRADLEY WITZEL: And how many ones?

AUDIENCE: Three.

DR. BRADLEY WITZEL: Three. Okay. But what's the reality if you know place value? How many ones are in that number?

AUDIENCE Two hundred forty-three...

DR. BRADLEY WITZEL: There are two hundred forty-three ones. How many tens?

AUDIENCE: Twenty-four.

DR. BRADLEY WITZEL: Twenty-four. If you understand decimals, its twent-four and three ten -- tenths, tens--oh, my gosh, my mind. And then how many hundreds? And I'll lose track, two and forty-three hundredths, hundreds. Why does that sound confusing? If you didn't get scientific notation in high school or college, that does sound confusing, but if you nail scientific notation in life, you're like, what's the big deal? Why does Bradley even trick around us? What's wrong with him? You're right. Shame on me, but I was not taught place value. I had to re-teach it to myself when I had children, I went, holy cow, I don't want -- teaching specialties for a long time because how do I make sure my children get what those children who are just as smart as my children, how do I make sure that I can preempt that? So here it is, some ways to help us out. If I do the race to a hundred games, race to tens? You put -- you put like a race to a hundred between two parkers and I can't walk around too often, so I'm sorry. I hope you [inaudible] across, but we'll -- we'll have a partnership game. So the cups between us, just says -- all right. This says tens and ones. They are based in blocks and two dice. So we roll them and I get -- I roll twelve -- you know what, this could be easier if I just -- if I just turn on -- and I roll a twelve. And after I roll this twelve, I got to put down twelve [inaudible] now, here is how you are going to read, because right now you're going -- you're probably thinking, "Okay. That makes sense but I'm probably beyond that. I don't know what to do with my students." So I roll twelve, we're going to lay down twelve. Then five -- bless you, two more. I roll down, I got twelve. Twelve ones is the same as one ten, two ones. Then I roll six more. So I lay down six more. And I ask the students, what did you get? He goes, we'll, I already got one ten, eight ones. And I said, or I have eighteen ones. Guys, this is a set up for decimals. I'm not as worried about this, I'm worried about the decimals later because right now, people are going to say .002 instead of its two hundredths. And by the way, teacher errors on decimals, they've done analysis of teacher error patterns and our teachers don't understand decimals [inaudible] it's like 24% errors of teachers doing decimals in our country. So we got to keep working on this as well. So when you're teaching your students, learn it ourselves. Never a cultural shift. So the same thing, you understand that if I write tens and ones here, if you're teaching decimals what could you add? Ones and tens and this could be eighteen tens or one and eight tens, it's the same number. The three is going to come up and stand in the three hundredth place...

FEMALE: Right.

DR. BRADLEY WITZEL: ...the zero has to understand [inaudible] to those zero tens and then the eight ones stands here or decimals, you can keep going with it. By the way, if you do the card game, where

they standard? Give commas out. Give a thousand comma, give the million comma. It's what we say. You do -- we do groups of hundreds based on comma. So two hundred and twenty-four thousand, right? So you need a person with that comma because that's the place -- time to say it. Just like you'll need a decimal point that says and, right? Or -- bless you, or a point. I mean, whatever you want to do with whom, but if you're going to do that, make sure you say those decimals correct afterwards. Other things that you all do. Going once--yes?

AUDIENCE MEMBER: Face value card game [inaudible] as the kids will be partnered, they each take turns pulling out the card. So if I say, like, they have to play like a thousand [inaudible] maybe ten thousand, and they have to put it down, the card from being at place when they put the number and -- so that the goal is who can beat who of getting closest to that number...

DRIVER. BRADLEY WITZEL: [inaudible] hundredths. And again card games -- and by the way, I worked some of the card counting in Vegas, card counting in Nevada. Getting those card games out there's good, all right? Someone's making money off that and it's not me. I'm the one that gives that money away. But the idea is making sure that we have an activity that forces interaction. And if you didn't hear that -- what he is describing with the card game, it's not what they can silently and passively work, it's forcing their language. Listen, these kids aren't going to go home and mom dad aren't going to have this languages with the, so we got to be consistent at school with it. And if you can't, send videos home, send cards home. Our -- my Third Grade -- my daughter's Third Grade teacher just sent home a list of how they're going to start saying numbers in class. And you should see -- for our family, it's like, woo hoo. I'm like -- I'm -- me, I'm not going to go--someone's finally going to understand my kid, right? So in that case, yeah, See 'n Say by the way is similar -- was it, Mike?

MIKE: Yes.

DR. BRADLEY WITZEL: Yeah. See 'n Say is similar what Mike was saying because one person gets the number written in numeral, and the other person in partner gets a number written long hand like, T-H-R-E-E-right? Four. Right? Thirty-four or, yeah, thirty, I wrote three, four, thirty four. And so they read through it and one has to go three tens four ones and the other person reads thirty-four and has to say three tens four ones. It's an interpretation gap. I'm not kidding. We took them all from the four language groups. All right. Place value. Get ready because it changes how you approach number. Remember when I said, you don't have to take seven always from the ones. You take seven for whenever you feel like taking seven. Math does not -- where students know all rules. That's not the way the numbers rules work. Same thing, three hundred forty-one, it depends on how you want to divide out, three hundreds, four tens and one to do that subtraction. It doesn't have to be what we're used to. You might've seen how I didn't spend much time at this at the -- at the other session, but this is twenty-six plus eighteen, so its two tens, six ones plus one ten, eight ones. Two tens plus one ten is three tens, six ones plus eight ones is fourteen ones. Then the interpretation comes out. So its four tens, four ones. Anybody do CRA? Anybody interested after the last session about CRA? Okay if you haven't heard of CRA, it is a -- I think I

have a little time to show some CRA. That was my goal. I'm going to show you addition, subtraction, multiplication, division with CRA, but it is a concrete way -- I use cups and sticks to show you how to approach Math problems. So in other words, you're going to tell me, how you have kids who can't solve these problems. And I'll say, "Have you tried some other ways to approach it?" And if there are four steps to solving it, there should be four concrete steps and four pectoral steps and we can't stop steps. All right. It works for addition it works for subtraction, and your standards, you can have it listed in there, all right? So, let me jump. We got ray models and ray models and ray models. [inaudible] time, I love [inaudible] I was a teacher who got upset when they took out the overheads in my classroom and they replaced it with nothing. They put a smart board in front. I got to tell you, I can't do live displays that way. You can't work with [inaudible] I don't have manipulatives up there, I'm trying to download pictures of manipulatives that my students are playing right now. I'm the teacher who said, I used to go to hotels even though -- do you still have a overhead here? A what? The dusty thing in the corner that people used for a stepping stool. Oh, I'm shocked. You know--and I'm--they were trying to revive the bull. So, let's get through some basics and just what is this cups and sticks way of showing these? The first one we talked about was addition. So that, when you show addition, take number line first and I think a show a progression with number lines there. But when I show a number line first, what am I going to do with this one? Which number do I start with?

AUDIENCE MEMBER: Three.

DR. BRADLEY WITZEL: I go -- well, I'm going to notice that I have three and then I'm going to count on. They're going to same -- they're going the same direction on the number line, so I get a three, four, five, right? And there's -- we might have miss earlier, I was trying to show the -- how did you get negative numbers happening so early? It's really not complex. Same direction on the number line, right? Three, four, five. So its -- like I said, its just directions. All negative numbers are just a direction. So please understand for this one, if I've got -- and I'm going to make it work nice and easy first, then we'll get more complex. But if I've got thirty-four and I'm trying to subtract twelve, or I've got three tens four ones and I'm trying to subtract one ten, two ones, do I have to start with the ones first? Start with the tens. Are these going the same or opposite directions on the number line?

AUDIENCE: Opposite.

DR. BRADLEY WITZEL: Opposite. So I go ten minus ten is zero, and then I'm going opposite directions here, four minus two is -- all right. Take out two from each. What's my answer? Positive two tens, two ones. Is that pretty easy?

AUDIENCE MEMBER: Uh-hmm.

DR. BRADLEY WITZEL: Okay. I'm going to make sure that I share how this works across. Let's take this one. What's happening here? I've got negative two tens, tho ones minus one ten, three ones. Are these numbers going on the same or opposite direction?

AUDIENCE: The same.

DR. BRADLEY WITZEL: So what am I doing? I'm going three and then I'm grouping these together as well. My answer, negative thirty-five or negative three tens five ones, right? Same direction. If that's confusing, then you just need to think of it this way. And then recognize we're going to the left on the number line. That's all it is. You're just -- so you're still going to the right, you don't even thinking about it. Working with accountants, there's always time for that. Just make sure I clarify why we do it this way. Are these going on the same or opposite directions on a number line?

AUDIENCE: Opposite.

DR. BRADLEY WITZEL: So what do I do? Ten minus ten, two minus two, what's my answer?

AUDIENCE: Negative.

DR. BRADLEY WITZEL: Negative eleven or negative one ten, one. I'm going to be messy. All right. Last one, I want to show it like this and we're -- I'm going to try to move faster. I've got thirty-one minus fourteen. I know, you want to -- I want to see a place value too I'm trying to be good for it. If I understand place value, how will I approach this? I'm going to say opposite directions. What am I doing?

AUDIENCE: [inaudible]

DR. BRADLEY WITZEL: Ten minus ten and then look over here. Icons. Wait, can I move it? I don't know if I can move this thing. Oh, look at you. All right. I'm getting there, technology, I'm an immigrant, but I'm catching up. I've got one minus four, what do I do? Take one away from each group. Now, you're ready to regroup and borrow. I love it. We all are. Let's stop. My answer is twenty minus three. Do you know how easy that is in kid's heads? And you're trying to say, now, take one ten -- listen, I've been a teacher. I did this. Well, we're going to do that and I'm going to take one and I'm going to exchange it, and I'll make -- you got that kid and he's like this, he's going to spend ten minutes on this subject because he's like, "Whoa, whoa, whoa, whoa, it's not perfect." And he's got like this and said, "Ah, ah, no, no, not yet." Now, I can exchange the three out, and you'll get seventeen. But I'm here to tell you, by the time you get here, that student's going to see twenty minus three and will be able to -- working concrete and will be able to say twenty minus three, huh? If he doesn't get it, yeah, you might be forced to exchange -- oh, why don't you just count backwards? Nineteen, eighteen, seventeen. If you don't like that, what I do, if you really think you have a student who has an issue with it, stack three on top, so he sees the difference and that's it. Well, I'm going to ask--we spend so much--why do teachers stop using manipulative objects in the classrooms? Cumbersome, messy, goofy, not always aligned to what I'm teaching. Well, yeah, but it wasn't the concrete objects' fault, it was how we are teaching's fault. And don't worry, I was that math professor who said, no, no. Exchange and we got it all perfect. And then I went to schools, tried this one, yes this is right. And finally, I asked a kid I go, "What's twenty minus three?" And he goes, "Seventeen." Okay. We're done. All right. Right down seventeen. Because I

wanted him to understand -- the purpose of the concrete was to show him that path of what we're doing. It wasn't to stop him or so. So far so good?

AUDIENCE MEMBER: Uh-hmm.

DR. BRADLEY WITZEL: Okay. Complaints about that, which I love. You guys are afraid of challenges, angry people. I just hate math. Why am I here? I want those kinds of complaints. I love it when people tell me, you know, I never use algebra, I don't know why we even teach it. I said, yeah, but your counting is algebra. The guy who runs your 401 (k) knows algebra. You might be interested in what he's doing with your money. I'm just guessing. Not that you can compute the interest of the loan you're paying right now. But that loan officer said, I'll give you -- it's three percent off, it -- what is it? Three thousand dollars or zero percent financing. If you make a mistake, the loan officer thanks you as he takes his commission, right? So which would you like to have? [inaudible] better for my account, right? You're not going to need to watch your money has already have an interest over the time that you're paying these loans, so, it's important that our students are financially secure enough to understand what they're doing. Anybody ever been to a car place and bought a car while calculating the interest of the person behind the computer? Have you ever seen a term the finance computer told you and said, "Here's what I'm doing." No, of course not. It's like buy or beware, dummy, I'm just giving you numbers. And I've been the guy who was sitting there for hours, near or two. So, please understand my take on this, I just want to be accurate. All right. So, this is one approach to it. I want to show you division because I did not show it earlier, and I want to make sure everybody sees what happens with division -- multiplication's pretty straightforward. I got to be cautious of -- I know I'm on video right now, so please understand what I'm using. These are bathroom glasses. It is a South Carolina version that I can't explain upfront that -- these [inaudible] some guy that's generic enough to get through. These are the acrylic bathroom glasses, yes. Yes, I bought them in Florida, so you can guess. But this is two groups of three. And what is two groups of three? Well, two groups of three means I've got one group of three, two groups of three. Two times three is equal to six. Multiplication calls for the unit, the whole unit to be there. Okay. My videographer's [inaudible] doing videos. I'm showing another one. And I'm not using large numbers the point will be made with small. I've got three times two, or I want them to think of this as three groups of two. Listen, if you really like equal signs and all that, you can say it is the same as or do a balance or whatever, but I'm just trying to simplify three times two. So, if it's three groups of two, that means each group has two. Three times two is equal to six. If you need to dump the cups because they're early in their understanding, dump the cups where they see there are sets, okay? But it is okay if they see that there are three groups of two and I -- multiplication now, I need to see the whole number. This is important because when you're dividing, it won't appear the same. Six sticks divided by three cups. What do we do here? I'm not going to do the cup game. I like that, but I'm not going to do that. Student's going to go through, and he's going to choose one at a time and he's going to go, "What's the answer?" I had six sticks divided by three [inaudible] cups. What's my answer? Two sticks per cup. We never got rid of sticks, we never got rid of cups. The only thing we moved are the--are the numbers, right? We

rearranged the numbers and kept the reference. So far good? Okay. It's important to understand that because students will try to think that things disappear when you divide and things naturally appear when you multiply. But instead, multiply you look at the group. Division, you look at each individual equal part. All right. Well, let's get a little ugly and [inaudible] so we have -- I'm not going to use halves because I can't break the sticks -- I'm not strong enough to do that. So, what we're going to do is seven divided by two -- so, seven divided by two -- let's go through -- what -- how do we do this? It's called partitive division. If you were in the last session, I showed some quick shots at what's called measurement division. This is partitive. This is for early childhood. It's trying to figure out what division means. So, it goes one, two -- oh, I forgot to use some of that. Okay. Three, four, five, six, seven -- oops, of course, he's going to do that. But what's the definition of division?

AUDIENCE: Equal parts.

DR. BRADLEY WITZEL: Equal parts. Are they equal?

AUDIENCE: No.

DR. BRADLEY WITZEL: Uh-oh. Let's go back. What's my answer? Three sticks per cup with one stick that needs to be broken into two cups. It's three and one half. What do you want me to really do? Cheap manipulatives, what you want to do with this? Of course, the reason why I don't often to do that is because I don't want my students to do that. So, I make my own rule. I'm the only one in the room allowed to break the sticks. That's my rule. And by the way, when I'm talking to school--high school, our original research, we never use popsicle sticks, we use toothpicks. And I didn't have [inaudible] problems in our rooms about pilots and a lot of settings, we have a problem, we said, mass it out there. Some classrooms were not as controlled as others and kids were practicing one-to-one correspondence. Okay. Okay. And so that's why we move to blunt objects. Okay. But these blunt objects are good blunt objects. They're also cheap. And my goal is to make sure that this is something we all can use. I mean, you don't have to buy. We've patented it. You know that patented this stuff? How can you patent it? We patented the maneuver of it. You know what we did? So that a publisher couldn't go out there and charge you a hundred twenty dollars from things you can buy at Walmart, I worked with a lot of engineers and they all got very upset because we got a phone call. Once the original research came out and it was through the roof, they go, "Oh, my gosh, this is the answer." Publishers started calling saying, oh, my gosh, we got to--we can go to the manufacture who made that, we'll get a hundred and twenty dollars from a glass can. [inaudible] what? For like bathroom little plastic cups and popsicle sticks? I said, "Yeah. But we make a plastic can pretty." And at that point is when we said, "Time to patent." And we patented it, so that so that no one could ever do it. You get me? You can do this. We want this for every teacher to be able to do, so that's why we--I love the videos, that's why my team loves you to see this. But understand its strengths and weaknesses. If you are stuck at concrete, you will be limited by the number of objects you can do at a time. Let's see. The next step is on because the goal of this is not to be really good at concrete. The goal of this is, so that one day I can say one, two, three, four, five, six, seven, eight, nine

divided into two different cups. Somebody who doesn't know what they're doing says, "Oh, my gosh, divided by zero, zero?" I even have teachers who get really nervous about this and they're drawing cups like this. Coffee mugs. Listen, whatever you need to do to get through the world, rock on. For me, they're the cups because a child is not seeing this until they have seen this. It's an adult issue, it's not a child's issue. So, in this case, you know what they end up doing? One, two, three, four -- I love this and eventually, a child's going to start trying to go larger groups. Listen, these are our kids who are copying things in the board, going R-U-N O [inaudible] and now we're going to start going, "No, no, no. Let's knockout two in a row, one, two. Let's knockout two in a row, one, two." And then I've got one left over. What's my answer? One, two, three, four, right? And then I've got one more that should be broken into two cups. The question was nine divided by two is equal to four and one half. Does that look -- does that look simplicity enough? I took some of these from Cecil Mercer's work and Cecil gave us some more from 1997, all we've done is take that initial model and work into a higher grade mark. So, this is some of the original stuff we did. If you missed the fractions stuff earlier today, shame on you but for us -- and these -- and that -- and then will be on video. You can watch that later. But this one I need you to see, some of the makings, sub or division builds to it. Remember, division does not make things go smaller. In multiplication, it doesn't make things get bigger. Multiplication means you're looking at the answers as the entirety and division means you're looking at each perfectly spaced [inaudible] for division. Are there questions on that [inaudible] other questions that I'm missing? So yes the time has set in. I noticed. I feel bad. Yes, yes? Okay.

AUDIENCE MEMBER: I didn't know if the question was back when they...

DR. BRADLEY WITZEL: Okay.

AUDIENCE MEMBER: [inaudible] well I'm working with students that are low [inaudible] in the Ninth Grade, [inaudible] the whole or negative or assume a plus, plus [inaudible] we're using a [inaudible] binds the negative or, [inaudible] plus negative

DR. BRADLEY WITZEL: I -- the reasons why you get concerned about this getting zipped and you really have a negative, positive of the question. That's what I'm doing now. The question was -- or adding -- we'll, was we're going to get your computation I think [inaudible] adding a negative, subtracting a negative, adding a positive, subtracting a positive. What's the best way to go about showing that? Now the reason why a lot of people wanna turn -- you're going to find some Middle School teachers from a while back are going to see this and say, "This is what's happening."

AUDIENCE MEMBER: Uh-hmm.

DR. BRADLEY WITZEL: The reason why they do this is, because in Physics these turn into vectors. I got to tell you, when I'm introducing this to Ninth and Tenth Grade I'm not worried about the next Physics professor.

AUDIENCE MEMBER: Uh-hmm.

DR. BRADLEY WITZEL: Okay? I love them [inaudible] I'm not saying they're not going to get it, I'm saying, I love the computation to be correct before I worry about them becoming a scientist. So, in that regard, my suggestion is, if I've got -- when I have a three -- wait, [inaudible] first, I have three minus a positive two. What do I do with this? So, I got a three and I got two signs next to each other. I'm subtracting a positive and I've got two. These two signs next to each other -- we're always trying to interpret them and know what's -- what's happening mathematically, it's not a cancel out. It's a multiplication problem. And really multiplying the negative one times the positive two, that's really all that's happening here. So, this is called a Field Model, in research it calls the Field Model, give the options. You're either going to be adding to from it or subtracting to it. Or what did they say I'm doing? Well, it says, I am not adding. Well, if I'm not adding, I must be subtracting. Does that make sense? Okay. So, let's get through some of those rules. I know we're used to the -- you see a two positive signs next to it. A good thing happens to a good person's a good thing. Right, we've all heard that.

AUDIENCE MEMBER: Absolutely.

DR. BRADLEY WITZEL: A good thing happens to a bad person is a -- oh, is that right? It's a bad thing. A good thing -- and why is that anyway? I'd probably compromise principles here. I'm not quite understanding. A bad thing happens to a good person is a bad thing and the one that I really get in trouble with my pastor, if a bad thing happens to a bad person it's a good thing. Yeah, that's -- I would turn that from Fox News -- I was going to have -- but in this case, while I'm going through this, I've got to check out. So, we have my symbols, if I have two positive symbols next to each other, I have to go and add it. I am subtracting. I am not subtracting. I am not adding. If you're the kind of person who wants things to be running across the number line of this, if it's positive three minus positive two, it means you start on positive three minus a positive two. That's what is happening. If it we're to be -- if it would be going to be like this, it means I've started a positive three minus a negative two. That's all that's happening. The reason why I don't do that well with Matchbox Cars and all that is the kid gets confused of when they turn and when they rock forward and backwards. So, I'm not saying that's not a good way to do it as a physical model, I'm just telling you that this Field Model seems to be -- I'm getting better results in the field model than I am on the number line for this. I'm not saying the number one is how the way to do it, because the rest of the topic would probably not going to finish through. It's got -- we have a lot in here. So -- so let's go through earlier today, I end up showing a progression using place value. This time, I'm going to try to show you well, both -- some basic facts. Is that an easily understood one? I hope you don't have to continue talking about why we need to practice multiplication. Does everybody agree that we need to practice multiplication on this country?

AUDIENCE: No.

DR. BRADLEY WITZEL: Our textbooks have been tagged internationally although lack of practice, that our kids get in this country. And they happen to do, "I'm the core textbooks, what to do?" They're still on the practice. The purpose of the Common Core is, it is longer benchmarked, and so they have more time to practice overtime. And what did they do? People are reinventing their old textbooks, signing Common Core labels on them and say, "Oh, Common Core, Common Core. Look, look, we've covered all the Math practices." No, what the point was is to get as more practice on the things that we never had time to practice before. So, there we go. Probably, something for the mouth for some of out there. You're going to see -- this is a PARCC Assessment. PARCC has done this, Smarter balanced is doing this. They all have this Fluency Assessments in the middle of the grade level made by the end of Third Grade, all math facts, right? All right. Given it for the number line result to come in, it's the curriculum is really getting filled-up with number lines for a good reason. We've talked about the incremental once versus the open kind of lines you cans see that, the US Department of head, this is the IES Practice Guide, it wasn't -- this is one of our practice guides here. This is two plus five. We don't -- I -- this -- I actually did the drawing. I don't like the [inaudible] things here but I can find a good icon that shows, go stop, go stop and whatever it is. If numbers are increased this way, it is a smooth from two plus five, two plus five. Two, three, four, five, six, seven, two plus five is seven. The other one is ten minus three. So, there are ten, right? Zero, probably, zero, ten, nine, eight, seven. Ten minus three is seven. And you've got to help them see this movement principle that's attacking all that number line. And then by already doing -- I don't know if you call it pyramid, the trees, the triangles. I've heard this called so many different ways, so, I'm trying to show you how firmly numbers work in here. All they're trying to do is, if you are showing this number, adding relationships also show them on the number line. Just match it up to a number line. I showed this one earlier one today. I apologize for the repeat. I just want to make sure it's the same point. When you're counting, I want to side this way. Just can't -- don't know how to do that on PowerPoint. But it goes zero to one, to two, to three, to four, to five. And you're trying to show that [inaudible] so then you see that this -- since I showed you this earlier today for some people. So I didn't explain it. Which is greater, five or three? Five is a larger number than three and the reason why you show simultaneous number lines is to make sure they see the length is different. Once they understood this magnitude, then they could do this. Which one is greater? Five or three? They can do this. This is how we usually start. You [inaudible] and everybody say, which one is greater? They don't know. So, we've got to make sure we show the length of these models before we show them doc number line models. Fair enough?

AUDIENCE: Yes.

DR. BRADLEY WITZEL: Your textbooks are not all equipped with this because they know -- textbooks are doing the same thing, well, these are good intention people, trying to fill the gap and fill for it. This is when Common Core came, they lost employees. It was no longer in Pennsylvania rep. So, they're having financial issues now. Why is that many of them have bankruptcy issues? What they're trying to do is patch the holes until we understand how Common Core 's going to work on them. All right. So, identify what one-third is on the number lines, same thing. Increments by thirds. More [inaudible] is one-

third. Now, let start to do with some computation on number lines as things go on. Two-thirds of one-half. I did all the odds early, right? So, that's two-thirds of one-half. I need to make sure where's one-half? Right there. What is two-thirds of the way of one-half? Well, break in two by -- right by the mere increment, break them up by thirds, identify where two of those groups would be. That's it. And are you doing this by the way? Bringing on the number lines [inaudible] those are the [inaudible] I'm like, "Okay. That's how I understand it." My point is -- this person has been attempting this. All right. So, same one -- let's -- oh, I don't -- I don't do division. We're out of time. You can think division of number lines as well but instead of -- this is like two-thirds divided by one-half, the question would be, how many one-halves go into two-thirds? Now, I know we're thinking in multiply or whatever I showed you earlier the actual knowledge. I was doing a conference with another country. So, the time zones are different. My daughter is at our window at the office like this, and she's like Fifth Grade, "What are you doing? What are you doing?" And so, at the end of the conference, I went upstairs, I lay down in bed with her and I said, "Oh, we're trying to figure out the number lines and how division works." And she goes, "Well, what do you mean?" I said to her -- and this is a dark room, I said, "Think of zero to one across here from the ceiling here. What is two-thirds divided by one-half?" She's, "What do you mean by it?" I mean, "How may one hands go into two-thirds?" And she goes, "Hmm, well, could be about..." now here out, "Is it about one and a half of them?" And I said, "You're pretty close." Think how many of our kids will come up with the answer of seven. What she's trying to mentally do is having one-halves in the number line can I take in this place to make that two-thirds? Does that make sense? What we're trying to do is let's try to help students understand that magnitude piece there. So, as to wrap things up, my biggest goal today, I'm hoping is that, you've seen some of these things that work out for Number Sense. I didn't have a word problems of scheme-based problem solving is a data ended up itself, it's practice with number lines and working through that. I wanted you to see the basics. Did everybody get at least the basics, the Number Sense and the components that we know if we do this early, they lead themselves doing on Algebra success. Fair? Did anybody get at least some of you certainly know that place value language? Good. How about the [inaudible]

AUDIENCE MEMBER: What other sample...

DR. BRADLEY WITZEL: I'm sorry?

AUDIENCE MEMBER: Concrete with the [inaudible]

DR. BRADLEY WITZEL: Concrete representational is the picture form, abstract. I'd probably try to write it up with CPA but I want nothing to do with those guys. So, concrete, picture and abstract [inaudible] my goal today is to make sure I've got two plans for you. One -- I'm just going to split, vertical planning, this needs to happen across grade levels but something we often forget is, if you have two teachers doing this in the third grade, make sure you have the same page with how you do this instruction, because in third grade, we may come out with equal scores, but we may mess up how fourth grade can have the problem. They at least need a baseline understanding before we can move on the alternative approaches. Fair

enough? All right. Conclusion. So, whatever that these tricks are that we've been taught, we've got to get beyond them. We've got to start undoing the tricks, finding out what made Math work the way it works and teaching our children. Listen, our goal, it's exact. We've missed out. Many of us have missed out in the best that Math can provide, but the next generations have that opportunity. Don't look at Common Core as the way that is going to keep us that.

AUDIENCE MEMBER: Yeah.

DR. BRADLEY WITZEL: Look at Common Core as a potential for change we can make of this country. All right? I hope that this helps. Did anybody learn at least one thing new?

AUDIENCE: Yes.

DR. BRADLEY WITZEL: All right. Well, thank you guys. I'm proud to see you. Thank you.